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The Influence of Environmental Variability and Media Coverage on the Dynamics of an Epidemic Model*

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Abstract Various infectious diseases seriously affect human health and social economy. It is a never-ending battle that human beings fight against infectious diseases. Media coverage has been an important weapon in virus war and has contributed to the epidemic prevention. In this paper, we focus on the dynamics of a stochastic SIRS epidemic model with media coverage and two mean-reverting Ornstein-Uhlenbeck processes. Firstly, we present the existence and uniqueness of the solution. Then, the sufficient condition for disease extinction is provided. In order to get the condition for disease persistence, we verify the existence of stationary distribution by constructing appropriate Lyapunov functions. Moreover, it is theoretically proved that the solution follows a normal probability density function around the endemic equilibrium of corresponding deterministic model. Finally, some numerical simulations are carried out to confirm theoretical results.

Keywords Media coverage, Ornstein-Uhlenbeck process, stationary distribution, density function

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1. Introduction

Infectious diseases seem to have become an unavoidable problem for mankind. The history of the development of human society is also a history of constant struggle against infectious diseases. In recent years, the outbreak of Ebola virus, Middle East respiratory syndrome (MERS), Corona-virus diseases 2019 (COVID-19) and other infectious diseases led to serious damage to human health worldwide. Several epidemics have brought great panic to the whole world, and even caused catastrophic consequences in many regions, such as economic recession and social shutdown. Therefore, a full understanding of transmission rules and control strategies of the

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disease is urgently needed. Mathematical modeling provides a reliable theoretical analysis for studying the pathogenesis of infectious diseases and predicting the development trend. It has become an effective tool to solve various phenomena and problems caused by the infectious disease.

For the epidemic dynamics model, Kermack and Mckendrick [1] proposed a landmark SIR epidemic model, which makes the epidemic model enter the era of quantitative analysis. After that, many authors [2–8] assumed that the total population $N(t)$ is divided into three categories at time t, including susceptible individuals $S(t)$, infected individuals $I(t)$ and recovered individuals $R(t)$. Recovered individuals in the SIR epidemic model are permanent immunity. But in fact, for most infectious diseases, such as cholera, influenza and malaria, acquired immunity wears off over time, which is a phenomenon well described by the SIRS infectious disease model. Ma et al. [2] considered that acquired immunity may disappear after a period of time and proposed a standard SIRS model:

$$\begin{cases} \dot{S}(t) = \mu - \bar{\beta}S(t)I(t) - \mu S(t) + \gamma R(t), \\ \dot{I}(t) = \bar{\beta}S(t)I(t) - (\lambda + \mu)I(t), \\ \dot{R}(t) = \lambda I(t) - (\mu + \gamma)R(t), \end{cases} \tag{1.1}$$

where μ is the natural birth and death rate coefficient. $\bar{\beta}$ denotes the average incidence rate. γ represents the immunity loss rate of recovered individuals. λ is the recovery rate of infected individuals. All the parameters are considered to be positive. Up to now, various versions of SIRS models have been studied and many research results have been achieved [6–11].

It is now widely noted that the mass media (television, Internet, microblog, Tik Tok, billboards and wechat) plays a key role in the spread of the disease [12–14]. For the public, people can keep abreast of the transmission route and the epidemic data through the media at any time. From the government's point of view, they can make full use of the powerful force of the media to publish essential disease prevention measures as soon as possible and broadcast the latest public health policies. The information reported by the media can affect the change of people's social behavior. People may go out less, receive vaccinations, self-isolation and wear masks, which will indirectly reduce the number of infected individuals or incidence rate among the population.

Research has found that the epidemic models with nonlinear incidence have more accurate and complex dynamics than those with bilinear incidence ($\bar{\beta}SI$) or standard incidence ($\bar{\beta}SI/N$) [14, 15]. The contact rates commonly used, bilinear and standard, cannot describe the impact of media coverage on the dynamics of the infectious disease well. Cai et al. [16] and Tchuenche et al. [17] held that the incidence rate in a model considering media coverage should be a monotonically decreasing nonlinear function with respect to $I(t)$. Li et al. [18] and Tchuenche et al. [19] constructed a modified nonlinear incidence rate $\beta(I)$ with media coverage:

$$\beta(I(t)) = \bar{\beta} - \frac{\bar{\beta}_e I(t)}{a + I(t)},$$

where $\bar{\beta}_e$ is the maximum reduced contact rate due to the presence of infected individuals. a is the half-saturation constant which reflects the impact of media