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Characterization of Distributions through Stochastic Models under Fuzzy Random Variables

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Abstract This paper is noteworthy because it investigates a novel method for comparing the expectations of stochastic models in fuzzy contexts. Actuarial science and economics both depend on stochastic models. Understanding the novel concepts of stochastic comparison of stochastic models based on the exponential order is the main advantage of this study. We solved the preservation properties and theorem, created a new definition, and put the fuzzy mean inactive time order definition into practice. Stochastic models are handled in a variety of applications.

Keywords Fuzzy set, random variables, stochastic orders

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1. Introduction

Applied probability, statistics, dependability, operations research, economics, and allied domains have demonstrated the value of stochastic ordering. A variety of stochastic ordering and related features have quickly emerged over the years. Let X be a nonnegative random variable that denotes the lifetime of a system with a distribution function F, survival function $\bar{F} = 1 - F$, and density function f. The conditional random variable $X_t = (X - t|x > t), t \geq 0$, is known as the residual life of the system after X_t, given that it has already survived up to X_t. The mean residual life (MRL) function of X is the expectation of X_t, which is given by

$$\mu_x(t) = \begin{cases} \int_t^{\infty} \left(\frac{f(x)}{\bar{F}(t)}\right) dt, & t > 0, \\ 0, & t \leq 0. \end{cases}$$

The MRL function is an important characteristic in various fields such as reliability engineering, survival analysis, and actuarial studies. It has been extensively

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studied in the literature, especially for binary systems, that is, when there are only two possible states for the system: either working or failing. The hazard rate HR function of X provides another useful reliability measure.

$$r_x(t) = \frac{f(x)}{\bar{F}(t)}, \quad t \geq 0.$$

The HR function is very helpful in characterising how the probability of witnessing the event varies over time and in identifying the appropriate failure distributions using qualitative data regarding the failure mechanism. The MRL function has been shown to be more effective in replacement and repair procedures, even though the shape of the HR function is still significant. The HR function only accounts for the possibility of a sudden failure at any given time. According to A. Arriaza, M.A. Sordo, and A. Surez Llorenz [1], there is a group of methods called transform stochastic orderings that help compare the remaining lifetimes and inactive periods at specific points. A new stochastic order, called the star order, was introduced. It is positioned between the convex order and the other two transform orderings. I. Arab, PE. Oliveira, and T. Wiklund [2] have developed a novel concept for the simple and thorough characterisation of instances in which one beta distribution is smaller than another based on the convex transform order. They find patterns in how likely it is for a beta-distributed random variable to be greater than its average or most common value. Arevalillo and H. Navarro [3] allow for stochastic comparisons of vectors with a multivariate skew-normal distribution. The new way of comparing is built on a specific change related to the multivariate skew-normal distribution and a common method used for comparing single skewed parts of that change. A. Arriaza, A. Crescenzo, M. A. Sordo, and Suarez-Llorens propose three functional measures of the shape of univariate distributions [4]. These metrics are appropriate with respect to the convex transform order. To close a gap in the literature, F. Belzunce, C. Martinez-Riquelme, and M. Perera [5] concentrate on giving sufficient conditions for a few well-known stochastic orders in dependability while handling their discrete forms. In particular, they discovered ways to compare two discrete random variables in certain stochastic orders by looking at the unimodality of the likelihood ratio. J.H. Cha and F.G. Badia [6] have studied the mean residual life, the bending property of the failure rate, the reversed hazard rate, and the mean inactive duration in mixtures. The idea of relative spacing's was first developed by F. Belzunce, C. Martinez-Riquelme, and M. Perera [7]. They demonstrate the relevance of this idea in several situations, such as economy and reliability, and we offer various results for evaluating relative spacing's among two populations. Belzunce, F., Ruiz, J.M., and Ruiz, M.C. [8] have compared organised structures formed from either a single group of parts or two distinct groups, based on various shifting and proportional stochastic orders. Izadkhah, S., and Kayid, M. [9] have proposed and explored a new type of stochastic order. Izadkhah, S., and Kayid, M. [9] look into important properties of the new stochastic order related to convolution, mixture, and shock model reliability methods. Xie, M., and Lai, C.D. [10] present an extensive summary of the theory and applications of dependence and ageing using mathematical methods for survival and reliability studies. A. Patra and C. Kundu [11] have enhanced the study of older properties of residual lifetime mixture models and stochastic comparisons. They looked at two different mixture models by using methods such as likelihood ratio, hazard rate, mean residual life, and variance residual life orders, along with two kinds of mixing distributions and