Quasi Statistical Convergence of Double Sequences in Neutrosophic Normed Spaces*

Chiranjib Choudhury[1,†] and Carlos Granados[2]

Abstract In this paper, we introduce the notion of quasi statistical convergence of double sequences in the neutrosophic normed spaces mainly as a generalization of statistical convergence of double sequences. We investigate a few fundamental properties of the newly introduced notion and examine the relationship with statistical convergence of double sequences in the neutrosophic normed spaces. In the end, we introduce the concept of quasi statistical Cauchy sequence of double sequences and show that quasi statistical Cauchy sequences for double sequences are equivalent to quasi statistical convergent double sequences in the neutrosophic normed spaces.

Keywords Quasi-density, quasi statistical convergence of double sequences, neutrosophic normed space

MSC(2010) 46S40, 03E72, 40G15.

1. Introduction

In 1951, the concept of statistical convergence was developed independently by Fast [3] and Steinhaus [22] to provide deeper insights into summability theory. Later on, it was further investigated from the sequence space point of view by Fridy [6], Šalát [19], Tripathy [23], and many researchers [1,7,11,13,17]. In 2003, Mursaleen and Edely [18] extended this concept over double sequences and mainly studied the relationship between statistical convergence and statistical Cauchy double sequences, statistical convergence, and strong Cesàro summable double sequences. Besides this, in [23], Tripathy studied various properties of the sequence spaces formed by statistical convergent double sequences and proved a decomposition theorem. Statistical convergence has many applications in various branches of mathematics such as number theory, mathematical analysis, probability theory, etc.

In 2012, Özgüç and Yurdakadim [16], extended the notion of statistical convergence to quasi statistical convergence using quasi-density. They further investigated the connection between quasi statistical convergence and statistical convergence. Following their work, several works have been carried out so far. In 2016, Ganguly

---

[†]the corresponding author.
 Email address: chiranjibchoudhury123@gmail.com (C. Choudhury), carlos-granadosortiz@outlook.es (C. Granados)
[1]Department of Mathematics, Tripura University (A Central University), Suryamaninagar-799022, Agartala, India
[2]Estudiante de Doctorado en Matemáticas, Universidad de Antioquia, Medellín, Colombia

and Dafadar [4] introduced and investigated the notion of quasi statistical convergence of double sequences as a natural generalization of statistical convergence of double sequences and studied some fundamental properties. For more details on quasi statistical convergence and its related developments, one can see the work of Özgüç [15] and Turan et al. [24], where many more references can be found.

The concept of fuzzy sets was first introduced by Zadeh [25] in the year 1965 which was an extension of the classical set-theoretical concept. Nowadays it has wide applicability in different branches of science and engineering. The theory of fuzzy sets cannot always cope with the lack of knowledge of membership degrees. To overcome the drawbacks, in 1986, Atanassov [2] introduced intuitionistic fuzzy sets as an extension of fuzzy sets. Intuitionistic fuzzy sets have been widely used to solve various decision-making problems.

Many times, decision-makers face some hesitations besides going to direct approaches (i.e., yes or no) in a decision making. In addition, we can obtain a tri-component outcome in some real events like sports, the procedure for voting, etc. Considering all in 2005, Smarandache [21] introduced the notion of Neutrosophic set as a generalization of both fuzzy set and intuitionistic fuzzy set. An element belonging to a neutrosophic set consists of a triplet namely truth-membership function (T), falsity-membership function (I), and indeterminacy-membership function (F). A neutrosophic set is determined as a set where every component of the universe has a degree of T, I, and F.

Kirişçi and Simsek [12] introduced the notion of neutrosophic normed space and investigated the notion of statistical convergence therein. Following their work, several researchers such as Khan et al. [8–10] and Şengül et al. [20] investigated various notions of convergence of sequences in the neutrosophic normed space. Recently, Granados and Dhital [5] have extended the above notion to the statistical convergence of double sequences in neutrosophic normed spaces. In this paper, using the double quasi-density we further extend it to quasi statistical convergence of double sequences. We also examine the relationship between statistical convergence and quasi statistical convergence of double sequences in neutrosophic normed linear spaces. Research on the convergence of sequences in neutrosophic normed linear spaces has not yet gained much ground and it is still in its infant stage. The research carried out so far shows a strong analogy in the behavior of convergence of sequences in neutrosophic normed spaces.

2. Definitions and preliminaries

Definition 2.1. [18] Let $K \subseteq \mathbb{N} \times \mathbb{N}$ and let $K_{m,n}$ denote the set

$$\{(i,j) \in K : i \leq m, j \leq n\}.$$

The double natural density of K is denoted and defined by

$$\delta^2(K) = \lim_{m,n \to \infty} \frac{|K_{m,n}|}{mn}.$$

Here, $|K_{m,n}|$ denotes the cardinality of the set $K_{m,n}$.

Definition 2.2. [18] A double sequence (x_{ij}) is said to be statistical convergent to l if for each $\varepsilon > 0$,