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Dynamics in a Delayed Eco-epidemiological Model with Disease in the Pest*

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Abstract In ecology, it is of great significance to research the influence of gestation period on the dynamics of eco-epidemiology. In the paper, we establish and explore a delayed predator-pest model with disease in pest. We first analyze the existence and local stability of each equilibrium of the model. Then, we investigate the existence of Hopf bifurcation at the coexistence equilibrium. Moreover, we calculate the normal form to examine the properties of Hopf bifurcation. Some numerical simulations are conducted to verify the theoretical results obtained and explore how the delay affects the biomass of pest. Our findings may contribute to a better understanding of the mechanisms of interaction between species in eco-epidemiology. At the same time, this study also provides an insightful perspective into the control of pests in ecosystems.

Keywords Predator-pest model, eco-epidemiology, gestation delay, Hopf bifurcation, normal form

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1. Introduction

In agriculture, organisms that cause harm to agriculture by destroying crops or parasitizing livestock are referred to as pests. They can directly or indirectly cause damage to the human population. It has been noticed that there are many animals and birds that feed on pests but do not affect the development of ecosystem, and therefore could be employed as an ecological method for the control of pest. This has been investigated extensively by constructing mathematical models in many recent research works [1–4]. On the other hand, some diseases may have a direct impact on pests, which can serve as a biological control to indirectly reduce pest populations [5–8]. In fact, once the pests are infected, they will become less mobile and their escape responses are weakened. Additionally because the lifestyle of the infected pests has changed and they live in habitats accessible to predators, and therefore can be easily caught by predators [9,10]. Thus, a profound comprehension of predation mechanisms is essential for elucidating the principles governing the eco-epidemiological system. Bhattacharyya et al. [7] proposed a mathematical model for pest management under virus infection which provides insights into the interaction between infected pests and predator in the epidemiological dynamics. Joly et al. [11]

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have demonstrated that severe infection of Echinococcus granulosus makes moose vulnerable to capture by wolves, suggesting that the disease plays a remarkable role in regulating moose populations.

Eco-epidemiology is considered as a very significant field of research in mathematical biology that simultaneously takes into consideration the ecological and epidemiological factors from both mathematical and ecological perspectives. The mathematical modelling researches based on the eco-epidemiological frameworks have attracted great attention of many researchers since the pioneering study of Hadeler and Freedman [12] and Chattopadhyay and Arino [13], and there are a large number of works devoted to exploring the impact of disease on ecosystems [14–16]. In eco-epidemiology, researchers explore an ecosystem with contagion either in predator [17–19] or in prey [20–22], or in both populations [23–25]. Li et al. [19] explored a mathematical model with diseased predators in an open environment, and obtained sufficient conditions for the disease to successfully invade the system. Hsieh and Hsiao [23] put forward an eco-epidemiology model with infected prey and infected predator, and their results indicate that the coexistence of organisms is determined by ecological threshold. It is worthy of noting that the eco-epidemiological models usually exhibit more complex dynamical behaviors than those without diseases.

Notice that time delay is an extremely common phenomenon in ecosystems; neglecting time delays implies neglecting the reality [26–28]. When developing an eco-epidemiological model, it is important to keep in mind that predators need to accumulate enough energy and nutrients over a period of time after eating to reproduce successfully, and therefore the reproduction is not instantaneous. It also important to explore the influence of time delay on the dynamics of the model [21, 29–31]. As an example, Xiao and Chen [21] proposed a delayed mathematical model with infected prey, in which the infected prey was assumed to be unable to reproduce and predators need certain time to reproduce after consuming the prey. Their results indicate that the delay may disrupt the stability of the system and lead to the occurrence of Hopf bifurcation. It is worth noting that the pregnant individuals may die during their pregnancy. It is necessary to incorporate this when constructing ecological dynamics models. To the best of our knowledge, few investigations have considered this factor in the eco-epidemiological models. In this paper, by explicitly incorporating the effct of this factor using a survival probability after pregnancy in the delay terms, we study how the time delay affects the dynamics of an eco-epidemiological model.

The organization of this paper is as follows. In Section 2, we present the model and some preliminary results, including the positivity and boundedness of the solutions. In Section 3, we perform the existence and stability analysis of equilibria of the model and derive conditions for the occurrence of Hopf bifurcation at the coexisting equilibrium. In Section 4, we compute the normal form to determine the nature of Hopf bifurcation. Some numerical simulations are given to confirm the theoretical results obtained in Section 5. Finally, we briefly summarize the paper in Section 6.

2. The model and some preliminaries

Assume that there is a disease spreading in a pest population, and that the infected individuals are too vulnerable to compete for the resources with susceptibles. Mean-