Journal of Nonlinear Modeling and Analysis
Volume 7, Number 4, August 2025, 1206–1239
https://journal.global-sci.org/jnma
DOI:10.12150/jnma.2025.1206

Fibonacci Wavelet Collocation Method for Solving a Class of System of Nonlinear Pantograph Differential Equations

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Abstract This paper introduces a unique strategy for solving numerically a class of nonlinear Pantograph differential equations using the Fibonacci wavelet collocation method (FWCM). First, we transform the nonlinear Pantograph differential equations system into a nonlinear algebraic system using this proposed approach. Next, the transformed nonlinear algebraic system is solved by using the Newton-Raphson scheme. The main advantage of this approach lies in its ability to reduce the computational complexity associated with solving Pantograph equations, resulting in accurate and efficient solutions. Comparative analyses with other established numerical methods reveal its superior accuracy and convergence rate performance. Further, a few examples are provided to evaluate the effectiveness of the suggested approach using absolute error functions. As far as our literature survey indicates, no one attempted the nonlinear Pantograph differential equations by FWCM. It compels us to study a system of Pantograph differential equations via FWCM.

Keywords Pantograph equations, collocation technique, Fibonacci wavelet, operational matrix of integration

MSC(2010) 34A08, 34A34, 34K28, 78A70, 93A30.

1. Introduction

Differential equations have been vital for defining and analyzing problems in many scientific disciplines for more than 300 years. The concepts of differential equations were initially introduced in the late seventeenth century by Gottfried Wilhelm Leibniz, Isaac Newton, and the Bernoulli brothers, Johann and Jakob. These happened naturally from these outstanding scientists' attempts to apply the new concepts of calculus to specific mechanical issues, such as the brachistochrone problem and celestial body motion routes. From the earliest ways of finding exact solutions in terms of elementary functions to the more recent methods of analytic and numerical approximation, their significance has inspired generations of mathematicians and other scientists to develop methods for investigating features of their solutions. Furthermore, they have been essential to the growth of mathematics since discoveries in analysis, topology, algebra, and geometry have frequently provided fresh insights into differential equations and since queries concerning differential equations have given rise to new fields of study.

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Systems of ordinary differential equations are often encountered in different science fields, such as biology, physics, economics, and engineering. Due to their versatility in modeling and describing dynamic processes, systems of ordinary differential equations (SODEs) find extensive use in diverse domains. Ordinary differential systems are essential tools to solve problems in the real world. ODE system of second order describes a broad range of natural phenomena. For instance, chemists can forecast reaction rates as well as the concentrations of reactants and products over time by using ODEs to simulate the kinetics of chemical processes, to comprehend and predict the behavior of economic variables like inflation, GDP growth, and investment. ODE systems are employed in financial modeling, and climate models use ordinary differential equations (ODEs) to forecast the earth's temperature, precipitation patterns, and other climatic variables throughout time.

The differential equations that describe the motion of a pantograph are crucial for understanding and analyzing the behavior of the mechanical linkage. These equations are significant for the following reasons: The differential equations make it easier to comprehend how the pantograph's orientation and position vary over time in response to outside inputs and forces. Forecasting and examining the pantograph's motion while it is in operation is necessary. Pantographs are sometimes employed with control systems to accomplish particular motion patterns or to follow a predetermined path. The differential equations play a crucial role in developing and applying control algorithms that regulate the pantograph's movement. The basics for developing mathematical models of pantograph systems are differential equations. Engineers can test theories, investigate alternative scenarios, and assess the pantograph's performance under various circumstances by using these models for simulation. Pantograph differential equations are significant because they are a vital tool for evaluating, planning, and enhancing the motion of pantograph systems in various applications.

Consider the following class of system of nonlinear Pantograph differential equations. [1]

$$\begin{aligned}
\mathscr{Z}_1'(\xi) &= \alpha_1 \mathscr{Z}_1(\xi) + f_1(\xi, \mathscr{Z}_i(\xi), \mathscr{Z}_i(q_j\xi)),\\
\mathscr{Z}_2'(\xi) &= \alpha_2 \mathscr{Z}_2(\xi) + f_2(\xi, \mathscr{Z}_i(\xi), \mathscr{Z}_i(q_j\xi)),\\
&\vdots\\
\mathscr{Z}_n'(\xi) &= \alpha_n \mathscr{Z}_n(\xi) + f_n(\xi, \mathscr{Z}_i(\xi), \mathscr{Z}_i(q_j\xi)).
\end{aligned} \tag{1.1}$$

$u_i(0) = u_{i_0}$, where $i, j = 1, \ldots, n$, $\alpha_i, u_{i_0} \in \mathbb{R}$, f_i are analytic functions, and $0 < q_j < 1$. $\mathscr{Z}_i(\xi)$ are dependent variables and ξ is an independent variable.

Multiple numerical approaches are created to approximate the solutions to those equations because of the challenges in finding the analytical solutions. Widatalla S. implemented the Laplace transform and Adomian decomposition method [1], Khuri S., [2] proposed the Laplace Adomian decomposition method, Cakmak M. et al. implemented the Fibonacci collocation method [3], He et al. proposed variational iteration method [4], Operational matrix approach based on Bernoulli polynomials was proposed by Rani D. et. al. [5], Odibat et al. developed Optimized decomposition method [6] and improved optimal homotopy analysis algorithm [7]. Davaeifar S., proposed the first Boubaker polynomials (FBPs) for the multi pantograph type equations [8].