Journal of Nonlinear Modeling and Analysis
Volume 7, Number 4, August 2025, 1254–1273
https://journal.global-sci.org/jnma
DOI:10.12150/jnma.2025.1254

Inertial Self-Adaptive Method for Solving Fixed Point Constraint Split Common Null Point Problem

Hammed Anuoluwapo Abass†

Abstract In this manuscript, we study the split null point problem in the settings of real Hilbert spaces using two different iterative methods. In our first method, we propose a self-adaptive algorithm with an inertial technique for solving split common null point problem and fixed point of a finite family of a demimetric mapping without the computation of the resolvent of a monotone operator. In our second method, we propose a self-adaptive algorithm with a multi-step inertial technique to approximate a solution of the aforementioned problems and to accelerate the rate of convergence of our iterative method. The selection of the stepsize employed in our iterative algorithms does not require prior knowledge of the operator norm. Lastly, we present a numerical example to show the performance of our iterative algorithms. The result discussed in this article extends and complements many related results in literature.

Keywords Fixed point problem, split common null point problem, demimetric mappings, iterative method

MSC(2010) 47H06, 47H09, 47J05, 47J25.

1. Introduction

Throughout this manuscript, let $\mathcal{H}$ denote a real Hilbert space with inner product $\langle ,.,\rangle$ and the induced norm $\|.\|$. Let I be the identity operator on $\mathcal{H}$, $\mathbb{N}$ be the set of all natural numbers and $\mathbb{R}$ be the set of real numbers. For a self-operator Ψ on $\mathcal{H}$, we denote by $Fix(\Psi) = \{p \in \mathcal{H} : \Psi(p) = p\}$, the set of all fixed points of Ψ.

Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be real Hilbert spaces and $B_j : \mathcal{H}_1 \to \mathcal{H}_2$ $(1 \leq j \leq m)$ be bounded linear operator. The split common null point problem (in short, SCNPP) is to find a point

$$x^* \in \mathcal{H}_1 \text{ such that } 0 \in \bigcap_{i=1}^{r} \Psi_i(x^*), \tag{1.1}$$

and such that the point

$$y_j^* = B_j x^* \in \mathcal{H}_2 \text{ solves } 0 \in \Delta_j(y_j^*),\ j = 1, 2, \cdots, m, \tag{1.2}$$

†the corresponding author.
Email address: hammedabass548@gmail.com, hammed.abass@smu.ac.za (H.A. Abass),
Department of Mathematics and Applied Mathematics, Sefako Makgatho Health Science University, P.O. Box 94, Pretoria 0204, South Africa.

where $\Psi_i : \mathcal{H}_1 \to 2^{\mathcal{H}_1}$ $(1 \le i \le r)$ and $\Delta_j : \mathcal{H}_2 \to 2^{\mathcal{H}_2}$ $(1 \le j \le m)$ are set-valued mappings.

The SCNPP (1.1)-(1.2) includes several optimization problems such as variational inequalities, convex feasibility problem and many constrained optimization problems as special cases, (see [7, 9, 24, 25, 29, 32]).

If $m = r = 1$, the SCNPP (1.1)-(1.2) reduces to the following split null point problem (in short, SNPP) which is to find a point

$$x^* \in \mathcal{H}_1 \text{ such that } 0 \in \Psi_1(x^*), \tag{1.3}$$

and the point

$$y^* = B_1 x^* \in \mathcal{H}_2 \text{ solves } 0 \in \Delta_1(y^*). \tag{1.4}$$

We denote by Θ the solution set of SNPP (1.3)-(1.4).

$$x^* \text{ solves } SNPP(1.3)\text{-}(1.4) \iff x^* = J_\lambda^{\Psi_1}(x^* - \gamma B_1^*(I - J_\lambda^{\Delta_1})B_1 x^*), \tag{1.5}$$

where $\lambda > 0$, $\gamma > 0$ and $J_\lambda^\Psi = (I + \lambda\Psi)^{-1}$ denotes the resolvent of a monotone operator Ψ.

In 2012, Bryne *et al.* [7] introduced the following forward-backward algorithm to solve SNPP (1.3)-(1.4): find $x_1 \in \mathcal{H}_1$

$$x_{t+1} = J_\lambda^\Psi(x_t - \gamma B^*(I - J_\lambda^\Delta)Bx_t) \tag{1.6}$$

where the stepsize $\gamma \in (0, \frac{2}{L})$ with $L = \|B^*B\|$, $J_\lambda^\Psi = (I + \lambda\Psi)^{-1}$ and $J_\lambda^\Delta = (I + \lambda\Delta)^{-1}$ are the resolvents of Ψ and Δ respectively.

Recently, Kazmi and Rizvi [22] studied the SNPP and fixed point of a nonexpansive mapping. They proposed the following iterative method to approximate the solution of the aforementioned problems as follows:

$$\begin{cases} y_t = J_\lambda^\Psi(x_t + \gamma B^*(J_\lambda^\Delta - I)Bx_t), \\ x_{t+1} = \alpha_t f(x_t) + (1 - \alpha_t)Sy_t, \end{cases}$$

where f and S are contraction and nonexpansive mappings respectively.

In 2018, Jailoka and Suantai [20] proposed the following iterative method for approximating the solution of SNPP and fixed point of a multivalued demicontractive mappings as follows:

$$\begin{cases} x_1 \in \mathcal{H}_1, \\ y_t = J_{\lambda_t}^\Psi(x_t + \gamma B^*(J_{\lambda_t}^\Delta - I)Bx_t), \\ u_t = (1 - \delta)y_t + \delta z_t, \ z_t \in Ty_t, \\ x_{t+1} = \alpha_t u + (1 - \alpha_t)u_t, \ t \in \mathbb{N}, \end{cases}$$

where γ, δ and the sequences $\{\alpha_t\}$ and $\{\lambda_t\}$ satisfy the following conditions:

(i) $\gamma \in \left(0, \frac{2}{\|B\|^2}\right)$ and $\delta \in (0, 1 - k)$,

(ii) $\alpha_t \in (0, 1)$ such that $\lim\limits_{t\to\infty} \alpha_t = 0$ and $\sum\limits_{t=1}^{\infty} \alpha_t = \infty$,