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Haar Wavelet Operational Matrix Approach for the Numerical Solution of Fractional Order Diabetes Mellitus Model

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Abstract Diabetes is a chronic disease which many people suffer from seriously. This study introduces a novel approach called the Haar wavelet collocation method (HWCM) to study the analysis and numerical approximation of the fractional order diabetes mellitus model. We built the operational matrix of integration (OMI) using the Haar wavelet to solve the diabetes mellitus model, a system of fractional differential equations. First, we transform the diabetes mellitus model into a system of algebraic equations using the operational matrix of integration of the Haar wavelet. The obtained system is further considered using the Newton-Raphson technique to extract the unknown Haar coefficients. Here, we use the calculus of fractional derivatives of a mathematical model to study and investigate the dynamic behavior of diabetes. We find numerical results for the validation of fractional order derivatives. Using the model parameter values, these numerical results are seen from both mathematical and biological perspectives. Numerical tables and graphical representations provide a visual presentation of the obtained results. The results of the developed method, the RK4 method, and the ND solver solution are compared. The numerical results show how highly accurate and efficient HWCM is in solving the fractional order diabetes mellitus model. Further, we show the method's efficacy and dynamics in various settings by performing simulations with parameter values. Mathematica, a mathematical software, has been utilized for numerical computations and implementation.

Keywords Caputo fractional derivative, collocation method, operational matrix of integration, Haar wavelet, fractional order diabetes mellitus model

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1. Introduction

Chronic metabolic diseases like diabetes mellitus continue to pose a serious threat to world health. It is a long-term metabolic disorder defined by persistently high blood glucose levels brought on by insulin production action insufficiencies or both. These abnormalities disrupt the metabolism of proteins, fats, and carbohydrates, highlighting the pivotal role of insulin as an anabolic hormone. Patients with diabetes have a higher risk of coronary artery disease and are four times more likely

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to have a stroke compared to individuals without the condition. Untreated diabetes poses several risks, including abnormalities in vision that can result in blindness, increased susceptibility to infections, and loss of consciousness. On the other hand, some people, especially kids with total insulin deficiency, might exhibit obvious symptoms like polyuria (excessive urination), polydipsia (excessive thirst), polyphagia (increased appetite), blurred vision, and unintentional weight loss. The International Diabetes Federation estimates that by 2040, there will be an additional 200 million people with diabetes mellitus worldwide from the approximately 415 million who had the condition in 2015 [1]. A wide range of genetic susceptibilities, including differences in genes related to insulin secretion, production, and regulation, are included in the etiology of diabetes mellitus. Diabetes mellitus is characterized by intricate disruptions in protein homeostasis, lipid metabolism, and glucose metabolism. Vast research efforts have been devoted to determining the etiology of diabetes mellitus, investigating the underlying pathophysiological mechanisms, and creating efficient treatment approaches as our understanding of the condition grows [7].

To find cures for the epidemics and plagues that have beset humanity, infectious disease modeling has attracted a lot of attention recently. In the literature, integer-order infection models have been the subject of numerous studies. Omame et al. [6], highlighted the importance of taking preventative measures for COVID-19 and Dengue co-infection in Brazil using an integer order model. An integer-order co-infection model of Dengue fever, Zika virus cholera and Buruli ulcer, syphilis, and HPV with optimal control was the main focus of the authors [3–6]. But the models listed above have drawbacks because they don't account for memory, an essential component of faithfully simulating real-world situations [2].

This article aims to present a mathematical formulation of the diabetes mellitus model, which has attracted significant attention from the scientific community. In the mathematical modeling of diabetes mellitus, blood glucose, insulin, and other relevant variables are modeled mathematically to understand and explain their dynamics. These models are useful for researching the illness's fundamental causes, evaluating different treatment approaches, and modeling its course. Diabetes mathematical models are helpful for research and clinical applications. They aid in comprehending the illness and provide direction for creating more potent treatment regimens.

Consider the following diabetes mellitus disease mathematical model in the form of a system of ordinary differential equations [8]:

$$\left.\begin{aligned}
\frac{d\mathcal{S}(t)}{dt} &= \gamma - \rho\mathcal{S} - \chi\mathcal{I} - \mu\mathcal{R},\\
\frac{d\mathcal{X}(t)}{dt} &= \eta(\mathcal{S} - \mathcal{X} - \mathcal{I} - \mathcal{R})\mathcal{X} - (\rho+1)\mathcal{X},\\
\frac{d\mathcal{I}(t)}{dt} &= \varepsilon\nu\mathcal{X} - (\rho+\chi)\mathcal{I},\\
\frac{d\mathcal{R}(t)}{dt} &= (1-\varepsilon\nu)\mathcal{X} - (\rho+\mu)\mathcal{R},
\end{aligned}\right\} \tag{1.1}$$

with the initial conditions: $\mathcal{S} = \mathcal{S}_0$, $\mathcal{X} = \mathcal{X}_0$, $\mathcal{I} = \mathcal{I}_0$, $\mathcal{R} = \mathcal{R}_0$.

The above set of non-linear equations is used to describe this complex model. This diabetes model is divided into four subclasses: susceptible individuals $\mathcal{S}(t)$, carrier infectious individuals $\mathcal{X}(t)$, infectious individuals $\mathcal{I}(t)$, recovered individuals $\mathcal{R}(t)$ and environmental bacteria concentration $\mathcal{N}(t)=\mathcal{S}(t)+\mathcal{X}(t)+\mathcal{I}(t)+\mathcal{R}(t)$ denotes the total population number at time t. The model parameters are as follows: