Journal of Nonlinear Modeling and Analysis
Volume 7, Number 5, October 2025, 1768–1791
https://journal.global-sci.org/jnma
DOI:10.12150/jnma.2025.1768

Modeling Divorce Dynamics Due to Extramarital Affairs with Piecewise Classical-Fractional Operator

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Abstract In recent decades, various methodologies have been proposed to model the complexities of challenging global problems across different domains. One such challenge involves understanding multi-step behaviors observed in certain situations. Newly proposed piecewise derivatives are known to address these issues. This study utilizes a mathematical model to examine the spread of a social issue of divorce among married couples resulting from extramarital affairs, using piecewise derivatives. Initially, we develop the model with Caputo fractional derivative and conduct some basic mathematical computations. Furthermore, the model is explored within the framework of the piecewise operator, incorporating both classical and Caputo operators. Within this framework, the study presents the existence and uniqueness of the solution using the fixed-point results. To analyze the behavior of the considered model, the Newton polynomial interpolation method is employed. The findings are subsequently illustrated through graphical representations, considering various values of fractional order.

Keywords Divorce, piecewise derivative, Caputo derivative, existence and uniqueness, numerical simulation

MSC(2010) 26A33, 34A08, 34A34, 37M05.

1. Introduction

Over the past twenty years, fractional calculus (FC) has gained popularity among researchers and scientists [7, 24, 25, 28, 44, 48]. FC applications cover a wide range of scientific and engineering fields, including theoretical, numerical, and experimental aspects [14, 32, 45]. Fractional differential equations (FDEs) are fundamentally characterized by their ability to incorporate memory effects. Classical models fall short compared with fractional models due to the inherent genetic properties, nonlocalities, and memory effects of fractional derivatives. The application of FDEs enhances our understanding of real-world phenomena, offering greater flexibility and precision in modeling. The literature extensively explores various fractional operators, each characterized by different kernels. Among these, the Caputo fractional derivative having a power-law kernel stands out as particularly significant

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due to its ability to account for memory effects and long-range dependence. The Caputo derivative integrates initial conditions naturally and is especially well-suited for applied fields. This derivative is widely used by researchers to solve various real-world problems and effectively captures how past interactions and events influence the current state of a system [2, 8, 9, 27, 30, 33, 34, 46, 47, 49–51]. Supporting the applicability of fractional derivatives, Atede et al. [6] analyzed the effectiveness of the Pfizer vaccination program against COVID-19 considering real data from Nigeria. They observed that varying fractional-order values exert different influences on each compartment of the model. Iwa et al. [19] proposed a Caputo-type new mathematical model to study co-infection dynamics of malaria and COVID-19. M. O. Olayiwola et al. [31] investigated a fractional model for COVID-19 that integrates high-risk quarantine measures and vaccination. Joshi et al. [20] proposed a four-dimensional system to estimate the effects of burned plastic and recycled plastic on air pollution.

Some real-world problems exhibit multi-step behaviors, where different processes occur in distinct phases. Such problems cannot be fully replicated using traditional fractional derivatives because the crossover behaviors between these phases remain inadequately addressed. To overcome this, Atangana and Araz [5] introduced a novel concept of piecewise derivatives and integrals. This new approach provides substantial advantages by allowing the analysis of mathematical models utilizing both classical and fractional operators inside a unified interval that can be subdivided. This significant development provides researchers with a powerful tool for investigating crossover behaviors in their studies. Piecewise derivatives have proven to be the most effective approach for illustrating this crossover event, as opposed to utilizing continuous derivatives in a range of real-world challenges [1,4,16,22,23,39–42].

Marriage is an officially and socially acknowledged tradition that is upheld by every religion to establish a relationship between a man and a woman (usually). Individuals enter marriage for numerous reasons, including the fulfilment of emotional and physical desires, familial and societal expectations, and raising children. From arranged or love marriages to now legally permitted same-sex marriages, several matrimonial trends have been noted over the years. Nowadays, a growing number of individuals are choosing divorce or seeking extramarital pleasures (infidelity), which leads to the deterioration of family structures. A successful marriage often requires effort and intentionality from both partners to maintain and nurture the relationship over time. This can be ascribed to a lack of patience and perseverance. Divorce is an endemic problem that, like any disease, has a significant impact on the social and economic structure of modern society. As a result, a structured methodical data analysis is required to tackle this socially complicated subject. In this world, individuals are generally expected to eventually encounter their life partner, whether through love or an arranged marriage. Additionally, some may seek emotional connections outside their primary relationship. As a result, they may face separation, which may lead to divorce in some situations. Certain cases of negotiations have also been recorded where spouses rejoin and give their marriage a second opportunity. To study this scenario, a mathematical model was formulated by Shah et al. [43] using ordinary differential equations.

The significance and dynamic nature of the divorce phenomenon have drawn attention from both biological and sociological perspectives, making it a subject of growing importance in diverse studies. Employing mathematical models, proven effective in controlling epidemics, can offer a valuable approach to proactively ad-