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An Analytical Technique on Numerical Solutions for EFKs of Fourth Order and Higher

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Abstract This study introduces an advanced analytical technique for solving the fourth-order Extended Fisher-Kolmogorov (EFK) equation, focusing on the application of Adomian decomposition methods (ADM) and modified Adomian decomposition methods (MADM) . The research outlines a systematic approach to deriving numerical solutions, facilitating the characterization and understanding of complex dynamics associated with the EFK equation. Additionally, the technique is generalized to higher-order extensions, enhancing its applicability in modeling various physical phenomena. The results illustrate the effectiveness of the proposed methods in achieving accurate solutions while addressing challenges inherent in higher-order differential equations.

Keywords Extended Fisher-Kolmogorov (EFK) equation, Adomian mechanism, initial conditions, higher order

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1. Introduction

The Extended Fisher-Kolmogorov (EFK) equation represents a significant evolution of the original Fisher-Kolmogorov equation, which models the dynamics of biological populations and diffusion processes. The EFK equation is characterized by its nonlinear nature and has been crucial for understanding complex phenomena such as pattern formation in biological systems, bistability in reaction-diffusion models, and other ecological interactions [14]. Mathematically, it describes how population densities evolve over time, incorporating higher-order spatial derivatives that facilitate the modeling of more intricate spatial dynamics [20, 21]. Despite its importance, solving the EFK equation poses substantial challenges due to its nonlinear characteristics and the complex initial conditions often involved in practical applications. Traditional analytical methods may not suffice, necessitating the exploration of alternative approaches for obtaining approximate solutions because they frequently struggle with the nonlinear characteristics of the EFK equation. These methods often assume linearity or rely on perturbative approaches that fall short when confronted with strong nonlinearities.

This is where the Adomian Decomposition Method (ADM), a powerful semi-analytical technique for solving a broad class of nonlinear ordinary and partial

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differential equations, developed by George Adomian during the late 20th century [1,3] becomes relevant.

In essence, ADM expresses the solution as a sum of functions, where each function corresponds to a term in the series. The nonlinearities in the equations are handled using Adomian polynomials, which facilitate the systematic computation of terms in the series expansion. This approach not only provides a straightforward means of addressing nonlinear equations but also enhances the flexibility in application to various fields, including physics, engineering, and applied mathematics.

Overall, the Adomian Decomposition Method stands out as an effective and accessible solution technique for tackling complex differential equations, making it a valuable tool in both theoretical and applied mathematics [4,6,12].

In another context, many modifications have been made to this method with the aim of improving it called Modified Adomian Decomposition Method (MADM). This technique provides a systematic framework for decomposing nonlinear problems into simpler components, enabling more manageable calculations and convergence to the true solution and it provides a structured way to address the nonlinear aspects of the EFK equation by decomposing the solution into an infinite series. [7,16].

MADM's ability to manage various forms of nonlinear dynamics makes it suitable for a wide range of applications beyond the EFK equation, including scenarios that feature complex boundary conditions or initial value problems. This adaptability allows researchers to apply the same technique across different problems without significant reforms to the method and has been successfully applied to a range of differential equations, showcasing its adaptability and robustness in tackling nonlinear dynamics look in [5,9,13,17]. The progression of differential equations from lower to higher orders, particularly the transition from fourth to sixth order, is rooted in the mathematical need to solve increasingly complex problems across various fields such as physics, engineering, and applied mathematics. High-order differential equations incorporate more derivatives and, consequently, provide richer models to capture the behavior of dynamic systems, wave propagation, thermal conduction, and other phenomena influenced by multiple variables. In the context of differential equations,"order" refers to the highest derivative present in the equation.

The extension from fourth to sixth order often involves sophisticated methods such as power series, Taylor expansions, and eigenfunction expansions. These techniques not only expand the class of functions that can be analyzed but also enhance the theoretical framework required for solving boundary value problems and initial value problems [15] . The need for higher-order derivatives arises in scenarios where the behavior of a system cannot be adequately described by simpler models. For instance, systems characterized by stiffness or complex interactions may necessitate these advanced formulations for accurate predictions and solutions. The exploration of higher-order differential equations reflects a broader trend in mathematics toward developing tools capable of addressing the multifaceted challenges posed by real-world applications [22].

In this paper, we aim to present a thorough investigation of the solution techniques for the EFK equation using the modified Adomian decomposition method. We first outline the theoretical foundations of the EFK equation, highlighting its relevance in contemporary research. Then, we detail the implementation of MADM, illustrating its effectiveness in deriving approximate solutions. Finally, we demonstrate the practical applicability of our findings through numerical examples,