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Controllability of Nonlinear Fractional Systems with Multiple State and Control Delays

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Abstract This study focuses on examining the controllability of nonlinear fractional differential systems in finite-dimensional spaces, considering multiple delays in both state and control. For linear systems, the necessary and sufficient conditions for relative controllability are established through the definition and application of the Gramian matrix. For nonlinear systems, controllability conditions are derived using Schauder's fixed point theorem.

Keywords Fractional differential system, relative controllability, multiple control delays, multiple state delays, Mittag-Leffler-type function, multivariate matrix equations

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1. Introduction

Fractional calculus constitutes a significant advancement over classical calculus, offering a wide range of essential functionalities that traditional calculus fails to adequately address. For example, numerous pressing societal concerns and empirical scientific challenges can be represented in a more coherent manner, particularly when accounting for the inherent uncertainties present in various dynamical systems. As a result, fractional calculus has become increasingly relevant across a multitude of disciplines, including mathematical physics, engineering, biophysics [15, 22, 28, 31, 53, 66, 67], nanotechnology applications [14], signal processing [54], circuit theory [57], and geophysical modeling such as earthquake analysis [27], among others.

Control theory is fundamentally grounded in the concept of controllability, which remains a cornerstone in the field of control systems. Several studies have addressed various aspects of controllability in semilinear and fractional dynamical systems. For instance, the study in [62] investigates the complete controllability of a semilinear stochastic system with multiple delays in control within a stochastic framework; however, it does not address fractional dynamics or delays in the state variables. Dauer and Gahl [17] established controllability results for nonlinear systems that incorporate delays. Balachandran and Dauer [7] conducted an in-depth analysis of controllability issues for both linear and nonlinear systems characterized by delays. Balachandran et al. [8,12] investigated the relative controllability of nonlinear fractional dynamic systems exhibiting both multiple delays and distributed delays in their control inputs. Klamka [10, 32] demonstrated controllability in linear and

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nonlinear systems influenced by time-varying delays. Recently, Mur et al. [49] investigated the relative controllability of fractional-order linear systems with delay components. The work [3] discusses the relative controllability of Caputo-type fractional systems involving a single state delay and multiple control delays; however, it does not address systems with Riemann-Liouville fractional derivatives, multiple state delays, or multivariate matrix-based formulations. The work in [61] investigates interior approximate controllability for second-order semilinear systems by reducing them to first-order systems and applying the Leray-Schauder alternative theorem. The study in [37] focuses on approximate controllability of fractional systems with a single state delay in Banach spaces using generalized Gronwall's inequality and compactness arguments, yet it does not handle multiple delays in control or noncommutative system structures.

It is a well-established fact within the realm of mathematical analysis that a differential delayed equation is fundamentally characterized by the incorporation of three essential components: the state of the system in the past, the state of the system at the present moment, and the corresponding rate of change of the system with respect to time. Prominent scholars Volterra and Minorskii have employed theoretical frameworks closely aligned with the structure of delay differential equations in various scholarly contributions, including, but not limited to, investigations into viscoelastic phenomena, predator-prey dynamics [68, 69], and applications in automatic steering and ship stabilization systems [47]. Within the context of these mathematical formulations, the concept of delays is intricately woven into the fabric of the states themselves, thereby adding complexity to both the analysis and interpretation of such equations. A substantial body of academic literature exists that delves into the intricacies of these specific types of differential delayed equations, as evidenced by the comprehensive studies referenced in [2, 4, 18–21, 23–26, 30, 36, 38–46, 50–52, 60, 70]. Nevertheless, it is noteworthy that a significant gap persists in the literature regarding the role of single and multiple delays in the context of control theory as it pertains to delay differential equations. Moreover, it is imperative to acknowledge that such equations have been extensively examined with respect to their controllability properties, as demonstrated by the contributions listed in [1, 5, 6, 10, 11, 13, 16, 17, 33, 34, 48, 59].

To avoid terminological ambiguity, we begin by explicitly defining the core notions employed in this study. Controllability refers to the ability to steer the state of a system from any initial state to any desired final state within a finite time interval using admissible control functions. Relative controllability is a weaker concept that concerns whether the system's state can reach a specific subspace or trajectory manifold, rather than the entire state space. Function controllability, in turn, pertains to the ability to control solution functions—defined over infinite-dimensional spaces such as $C([a, b], \mathbb{R}^n)$—rather than finite-dimensional state vectors.

In the context of these complex equations, it becomes crucial to draw a clear distinction between the concepts of function controllability and relative controllability within the framework of Euclidean space. This differentiation is particularly salient because, despite the fact that the solutions derived from these equations manifest as trajectories within the confines of Euclidean space, the inherent and more appropriate "state space" that governs their behavior is, in reality, a function space, which necessitates a more nuanced understanding. For the specific aims and objectives of this scholarly inquiry, we intentionally restrict our analysis to the concept of relative controllability alone. In addition, it is essential to recognize that, unlike in classical