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# Some Results on Fractional Corrected Euler-Maclaurin-Type Inequalities Related to Various Function Classes

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**Abstract** In this paper, some corrected Euler-Maclaurin-type inequalities are established by using various function classes involving Riemann-Liouville fractional integrals. We then present our findings using examples and special cases of the theorems that we have discovered. Moreover, we provide several fractional corrected Euler-Maclaurin-type for bounded functions. Additionally, for Lipschitzian functions, we create a few fractional corrected Euler-Maclaurin-type inequalities. Lastly, we provide some fractional corrected Euler-Maclaurin-type inequalities for functions of bounded variation.

**Keywords** Quadrature formulae, Maclaurin's formula, convex functions, fractional calculus

**MSC(2010)** 26D07, 26D10, 26D15, 65D32.

## 1. Introduction

Inequality theory is well-known and still an interesting area of research with a wide range of applications in many fields of mathematics. Convex functions are also significant in the theory of inequality. Afterwards, mathematicians in the mathematical sciences have become interested in fractional calculus due to its fundamental properties and its applications. Because of the importance of fractional calculus, mathematicians have studied a number of fractional integral inequalities.

Dragomir [1] provided an estimate of remainder for Simpson's quadratic formula in the case of bounded variation functions, with applications in the theory of special means. Furthermore, a number of fractional Simpson-type inequalities for functions with convex second derivatives in absolute value were demonstrated in article [2]. Furthermore, in the domain of differentiable convex functions, Budak et al. [3] examined a number of variants of Simpson-type inequalities using generalized fractional integrals. For additional information on Simpson-type inequalities and other characteristics of Riemann-Liouville fractional integrals, readers can see [4, 5] and their references. In the literature, evaluations for three-step quadratic kernels are sometimes referred to as Newton-type results because the three-point Newton-Cotes

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quadrature corresponds to Simpson's second rule. A great deal of research has been done on Newton-type inequalities. For instance, a number of fractional Newton-type inequalities were reported in article [6] for the case of bounded variation functions, and several Newton-type inequalities were proved by using the Riemann-Liouville fractional integrals for the case of differentiable convex functions. Later, various Newton-type integral inequalities were established by Erden et al. in paper [7] for functions whose first derivative is arithmetically-harmonically convex in absolute value at a certain power. According to Sitthiwirattham et al., some fractional Newton-type inequalities for constrained variation functions were given by [8]. Furthermore, Gao and Shi [9] proposed a new convexity-based Newton-type inequality and provided specific applications for specific real function scenarios. Please refer to [10–12] and their references for more information on convex differentiable functions and other Newton-type inequalities.

Dedić et al. [13] created a set of inequalities using the Euler-Maclaurin-type inequalities, and the outcomes were used to produce specific error estimates in the case of the Maclaurin quadrature rules. To establish a set of inequalities, the Euler-Simpson 3/8 formulas were also used. The findings were used to provide some error estimates for the Simpson 3/8 quadrature rules in article [14]. Additionally, several Euler-Maclaurin-type inequalities were stated in [15]. Later, some various corrected Euler-Maclaurin-type inequalities were proved using the Riemann-Liouville fractional integrals in paper [17]. The reader is referred to [18–27] and the references therein for further information on these kinds of inequalities.

This paper uses Riemann-Liouville fractional integrals to derive Corrected Euler-Maclaurin-type inequalities for a variety of function classes. A basic definition of fractional calculus and more research in this area are given in Section 2. We shall demonstrate an integral equality that is necessary to prove the article's primary conclusions, which are presented in Section 3. Moreover, Section 4 provide some corrected Euler-Maclaurin-type inequalities for a number of differentiable convex functions using the Riemann-Liouville fractional integrals. We shall provide some corrected Euler-Maclaurin-type for bounded functions by fractional integrals in Section 5. For Lipschitzian functions, some fractional corrected Euler-Maclaurin-type will be established in Section 6. The corrected Euler-Maclaurin-type will be demonstrated using fractional integrals of bounded variation in Section 7. Moreover, in Section 8 we will offer a number of graphical illustrations to show the accuracy of the recently established inequalities. We will talk about our thoughts on corrected Euler-Maclaurin-type inequalities and how they might affect future directions for study in Section 9.

## 2. Preliminaries

The Riemann-Liouville integrals $J_{a+}^{\alpha}f$ and $J_{b-}^{\alpha}\, f$ of order $\alpha>0$ with $a\geq 0$ are presented by

$$J_{a+}^{\alpha}f(x)=\frac{1}{\Gamma(\alpha)}\int_{a}^{x}(x-t)^{\alpha-1}f(t)dt,\quad x>a$$

and

$$J_{b-}^{\alpha}f(x)=\frac{1}{\Gamma(\alpha)}\int_{x}^{b}(t-x)^{\alpha-1}f(t)dt,\quad x<b$$