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A New Study on Common Fixed Points for (α-β-F)-Contraction Mappings

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Abstract This paper explores the existence of common fixed points for a newly introduced class of mappings called (α-β-F)-contraction mappings in metric spaces. We establish several common fixed point theorems under specific conditions, incorporating (α-β-F)-weak contractions to extend and generalize existing results in fixed point theory. Unlike many classical approaches, our framework retains flexibility while accommodating key structural properties such as compactness and continuity. To support the theoretical findings, illustrative examples are provided. These results enhance the understanding and applicability of fixed point theory.

Keywords Common fixed point, (α-β-F)-contraction, (α-β-F)-weak contraction

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1. Introduction

Banach's fixed point theorem for contraction mappings is a foundational result in mathematical analysis, particularly within metric fixed point theory. The Banach contraction principle [1] stands as a significant result, underpinning various approaches to ensuring the existence and uniqueness of solutions to a wide range of nonlinear problems, including differential and integral equations, optimization challenges, and variational inequalities. Numerous extensions and adaptations of this principle have been proposed, broadening its applicability. In 2012, Wardowski [2] introduced the concept of F-contractive mappings in metric spaces, establishing a fixed point theorem for such mappings in complete metric spaces. Subsequently, Wardowski [3] expanded on these ideas by defining F-contraction and F-weak contraction, which provide a substantial generalization of Banach's contraction principle. Further contributions to this field include the work of Gopal et al. [4], who introduced α-type F-contractions for self-mappings in complete metric spaces,

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thereby establishing conditions sufficient for the existence and uniqueness of fixed points. Additional generalizations can be found in works such as [5–12]. In 2013, Cicilline [13] presented an iterated function system of F-contractions, and in 2016, he explored weak F-contractions along with related fixed point results [14]. In 2018, Wardowski [15] further applied F-contractions to solve existence results, and in 2020, Popescu et al. [16] investigated two fixed point theorems concerning F-contractions in complete metric spaces. In 2024, Al-Salehi et al. [17] conducted a study on fixed point theorems related to extension and modified extension of α-F-contractions. In 2019, Asif et al. examined F-contractions and common fixed point theorems, focusing on their applications [19]. In 2021, Lucas and Santosh presented a common fixed point theorem for generalized F-Kannan mappings within metric spaces, accompanied by its applications [20]. In 2022, Gautam et al. [21] explored the existence of common fixed points for Kannan F-contractive mappings. In 2023, Wangwe investigated fixed point and common fixed point theorems for (γ, s, q)-F-contraction mappings [22]. Between 2023 and 2024, Raji et al. studied coincidence and common fixed points for F-contractive mappings, including those for fuzzy F-contractive mappings [23, 24]. Recently in 2024, Kanthasamy et al. focused on common fixed point theorems for (ψ, F)-contraction mappings [25].

2. Preliminaries

Definition 2.1 ([2]). Let $\mathcal{F}$ represent the family of all functions $F : (0, \infty) \to \mathbb{R}$ that satisfy the following conditions:

(F1) F is strictly increasing; that is, for all $\alpha, \beta \in (0, \infty)$, if $\alpha < \beta$, then $F(\alpha) < F(\beta)$;

(F2) For any sequence $\{\beta_n\} \subset (0, \infty)$, we have $\lim_{n\to\infty} F(\beta_n) = -\infty$ if and only if $\lim_{n\to\infty} \beta_n = 0$;

(F3) There exists $k \in (0, 1)$ such that $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.

Definition 2.2 ([2]). Let (X, d) be a metric space. A mapping $T : X \to X$ is defined as an F-contraction on (X, d) if there exists a function $F \in \mathcal{F}$ and a constant $\tau > 0$ such that for all $x, y \in X$, the inequality

$$\tau + F(d(Tx, Ty)) \leq F(d(x, y))$$

holds whenever $d(Tx, Ty) > 0$.

Example 2.1 ([2]). The following functions are elements of the family $\mathcal{F}$:

(i) $F(\alpha) = \ln(\alpha)$;

(ii) $F(\alpha) = \frac{-1}{\sqrt{\alpha}}$;

(iii) $F(\alpha) = \alpha + \ln(\alpha)$.

Definition 2.3 ([18]). Let $S, T : X \to X$ and let $\alpha : X^2 \to [0, +\infty)$. The mappings S and T are called α-admissible if, for all $x, y \in X$, the condition $\alpha(x, y) \geq 1$ implies that either $\alpha(Sx, Ty) \geq 1$ or $\alpha(Tx, Sy) \geq 1$.