Journal of Nonlinear Modeling and Analysis https://journal.global-sci.org/jnma
Volume 7, Number 6, December 2025, 2285–2299 DOI:10.12150/jnma.2025.2285

Exact Soliton of Fifth Order (1+1) Dimensional Triple Non-Linear Partial Differential Equations on Modified Truncated Expansion Methods

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Abstract In this manuscript, modified truncated expansion method-I taking the traveling wave variable $u(x,t) = y(r) = y(kx+wt)$ and modified truncated expansion method-II introducing the traveling wave variable $u(x,t) = y(r) = y(kx - wt)$ are built up to obtain analytical solution in the form of traveling wave solutions with different frequencies and velocities that can be constructed for triple (1+1) dimensional nonlinear partial differential equations (NLPDEs) such as Sawada-Kotera equation (SKE), generalized Korteweg-de Vries equation (GKdVE) and Kaup-Kuperschmidt equation (KKE), which have been widely used in mathematical physics. The present topic minimizes the complex nature and non-integrable characteristics to obtain solutions of NLPDEs. To demonstrate the influence of the parameters, 3D plots are generated for triple NLPDEs. This content is employed in physics such as magneto sound in plasma and nonlinear optics.

Keywords Modified truncated expansion methods, triple nonlinear partial differential equations, magneto sound in plasma, nonlinear optics, traveling wave solutions

MSC(2010) 35Q51, 35Q53.

1. Introduction

NLPDEs are highly helpful in a variety of domains, including fluid dynamics, water surface gravity waves, electromagnetic radiations, and ion acoustic waves in plasma. Several techniques are adopted to search the analytical solitons to NLPDEs including methods as found in [1–4]. The propagation of waves and sound in flat surface such as capillary-gravity and shallow water and magneto sound in plasma [5] are based on modelling of GKdVE, as described in equation (1.2) in Mathematical Physics respectively. The integrable fifth-order KdVE and the necessary conditions for the nonlinearity and dispersion parameters are investigated by Hirota method [6]. Later [2] invented stationary wave solutions of binary waves for fifth-order KdVE as results of Kudryashov-expansion and sine-cosine function techniques. In order to obtain two kinds of approximate solutions a simple standard truncated expansion approach (SSTEA) has been suggested for fifth-order GKdVE [7]. The Bäcklund transformation by new GKdVE is elaborated in [8] to extract the bilinear form as

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well as structural wave phenomenon of the governing model on account of gravity field in shallow water to carry out the propagation of long waves. The exponential stability results by suitable, Lyapunov functionals on KdVE with time-dependent delay on the boundary are very much attracted in [9]. The suitable transformation is taken in [10] to reduce the GKdVE to a quadratic ordinary differential equation by implementing the new version trial equation approach for finding new solitary wave solutions. To estimate the movement of water waves the sech method is established to explore novel analytical traveling wave solutions for a nonlinear KdV in [11]. The exp-function and modified exp-function methods are implemented in [12] to explore solitary, peakon, periodic, cuspon and kink wave solutions of KdVE. Authors [13] applied the natural decomposition method for solving the time-fractional coupled KdVE. Numerical soliton KdVE in infinite dimension using the steepest descent method is found in [14]. The numerical solution of KdVE by [15] is based on the trapezoidal and implicit mid-point method via Pade technique.

The various mathematical physics concepts like quantum, optics and travelling wave solution in shallow water are mainly focused on modelling of SKE. The Hirota bilinear techniques [16,17] and modified auxiliary equation technique (MAET) and the extended direct algebraic technique (EDAT) in [18] are adopted for different analytical solitary solitons of fifth order SKE. Further, Hirota direct method [19] is investigated to exploit the -soliton solutions like, single soliton, double soliton and triple soliton solutions of the integral SKE. The independent transformation by [20] is implemented for multidimensional KdV-Sawada-Kotera-Ramani equation (KdVSKRE). The Exp-function technique has been offered to find analytical solutions of generalized SKE in [21]. The tanh-coth method is imposed on SKE to achieve explicit exact soliton [22].Yang transform (YT) with the Adomian decomposition method (ADM) and homotopy perturbation technique (HPT) are used on the seventh-order time-fractional Sawada-Kotera-Ito problem (TFSKIP) to obtain the solution in [23]. The KKE is implemented in physical sciences in different approaches like plasma physics, nonlinear optics and fluid dynamics. The analytical solution of traveling wave of KKE is found by using the Fan sub-equation method [24]. The nonlinear evolution equation particularly time fractional KKE is solved in [25] to obtain new general forms of analytical solutions by employing improved and extended $\dfrac{G'}{G}$technique that would applicable in mathematical modeling of nonlinear form. The analytical solution of KKE is extracted by implementing double $\left(\dfrac{G'}{G}, \dfrac{1}{G}\right)$ expansion approach in [26]. The authors in [27] have shown their interest towards homotopy perturbation transform method (HPTM) and Yang transform decomposition method (YTDM), to solve the fractional nonlinear seventh-order KKE through the Caputo operator. The homotopy method is addressed in [28,29] to investigate the numerical solution of KKE. Further, the one method is synthesized in [30,31] to get analytical soliton of (1+1)-Dimensional KKE. In addition to these, several computational approaches may be employed to approximate soliton solutions, including the modified auxiliary function approach [32], the natural decomposition method with Atangana-Baleanu derivative in Caputo manner and Caputo-Fabrizio [33]. There are several methods [[34–53], [61–65], [59], [60]]. Kudryashov and functional variable are implemented to obtain the analytical solutions of the strain wave equation for briefing wave propagation in micro structured solids in [54]. The soliton wave for nonlinear Schrödinger equation is also