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Application of the Functional Variable Method to Some Nonlinear Evolution Equations

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Abstract The functional variable method is a highly effective approach for deriving exact solutions to nonlinear evolution equations. It offers broad applicability in addressing nonlinear wave equations. In this paper, the functional variable method is employed to obtain soliton solutions for the Radhakrishnan-Kundu-Lakshmanan (RKL) equation and the Landau-Ginzburg-Higgs equation. Exact solutions play a crucial role in uncovering the internal mechanisms of physical phenomena. Graphical representations of the obtained optical soliton solutions are provided to illustrate some of their physical parameters.

Keywords Solitary wave solution, Radhakrishnan-Kundu-Lakshmanan equation, Landau-Ginzburg-Higgs equation, functional variable method

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1. Introduction

The solitary wave solutions to nonlinear evolution equations (NLEEs) are highly significant due to their potential applications in various physical fields, including neural physics, chaos theory, diffusion processes, and reaction dynamics, among others. The solitary wave solutions to nonlinear evolution equations (NLEEs) in mathematical physics have seen significant advancements over the past decades. The direct pursuit of exact solutions for PDEs has gained increasing interest, partly due to the availability of computer algebra systems like Maple and Mathematica, which facilitate complex and labor-intensive algebraic computations. There are several schemes, for instance, Haar wavelet method [1], homotopy asymptotic method [2], extended tanh technique [3], variational iteration method [4] the modified Kudryashov method [5], the auxiliary equation method [6, 7], the (G'/G)-expansion method [8–10], the first integral method [11, 12], the new generalized (G'/G) expansion method [13], the improved Kudryashov method [14], the tanh-method [15, 16], the extended trial equation method [17], the (G'/G, 1/G)-expansion method [18], the sine-Gordon equation expansion method [19], the improved F-expansion method [20], the modified simple equation method [21, 22], the improved Bernoulli sub-equation function method [23], the Darboux transformation method [24–26], the Modified Sub-Equation method [27], the Sardar sub equation method [28], etc. which have been used for searching stable closed-form wave solutions to NLEEs.

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The Radhakrishnan-Kundu-Lakshmanan (RKL) equation is a generalized form of the nonlinear Schrödinger equation that describes the dynamics of soliton propagation in polarization-preserving fibers. This equation was first introduced in 1999 [29]. Over the years, it has been extensively studied by various researchers. For instance: Arshed et al. [30] derived optical solitons for the RKL equation with full nonlinearity. Biswas et al. [31] analyzed the RKL equation using the extended trial function scheme. Gaxiola and Biswas [32] explored the RKL equation via the Laplace-Adomian decomposition method. Ghanbari et al. [33] obtained exact optical solitons of the RKL equation under Kerr law nonlinearity. Ganji et al. [34] examined the nonlinear RKL equation in detail. Elsherbeny et al. [35] applied the improved modified extended tanh-function method to study the RKL equation. These studies highlight the diverse mathematical approaches and the significance of the RKL equation in understanding soliton dynamics.

The Landau-Ginzburg-Higgs equation, developed by Lev Davidovich Landau and Vitaly Lazarevich Ginzburg, has broad applications in explaining phenomena such as superconductivity and drift cyclotron waves in radially inhomogeneous plasma, as well as coherent ion-cyclotron waves. Various methods have been employed to derive unique soliton solutions for the integrable nonlinear evolution equation (NLEE). For example, Bekir and Unsal [36] applied the first integral method to analyze Landau-Ginzburg-Higgs equation and obtained exponential function solutions. Iftikhar et al. [37] explored different types of analytic solutions using the (G'/G,1/G)-expansion method, yielding general soliton solutions and kink-shaped solitons under varying parametric conditions. Islam and Akbar [38] utilized the IBSEF method to derive several stable solution types. Ahmad K et al. [39] used power index method to derive exact solutions of Landau-Ginzburg-Higgs equation. These studies demonstrate the versatility of mathematical approaches used to analyze the Landau-Ginzburg-Higgs equation and its relevance in physical applications.

The powerful and effective method for finding exact solutions of nonlinear evolution equations was proposed in [40, 41] by Zerarka et al., which is called the functional variable method. Recently, Babajanov [42–44] applied this method to obtain soliton solutions of various NLEEs.

In this paper, we extend the application of functional variable method to solve the Radhakrishnan-Kundu-Lakshmanan (RKL) and Landau-Ginzburg-Higgs equation. In Section 2, we propose the basic idea of the method for finding exact travelling wave solutions of NLEEs. In Section 3, we establish the exact travelling wave solution for Radhakrishnan-Kundu-Lakshmanan (RKL) and Landau-Ginzburg-Higgs equation. Finally, in Sections 4 and 5, graphical representation of the equations and conclusions are given.

2. Basic idea of the Functional Variable Method

Consider the nonlinear partial differential equation(NLPDE) of the form:

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{xy}, u_{xt}, u_{yt}, u_{yy}...) = 0, \tag{2.1}$$

where P is a polynomial in $u = u(x, y, t)$ and its partial derivatives. The main steps of this method can be described as follows:

Step 1. In order to find the travelling wave solution of Eq. (2.1), we introduce the wave variable $\xi = ax + by - ct, a, b, c = const.$, so that

$$u(x, y, t) = U(\xi), \tag{2.2}$$

Then the NLPDE is transformed to an ordinary differential equation (ODE) as:

$$Q(U, U_\xi, U_{\xi\xi}, U_{\xi\xi\xi}, ...) = 0, \tag{2.3}$$

where, Q is a polynomial in $U = U(\xi)$ and $U_\xi = \frac{dU}{d\xi}, U_{\xi\xi} = \frac{d^2U}{d\xi^2}$ and so on. If all terms contain derivatives, then Eq. (2.3) is integrated where integration constants are considered zeros.

Step 2. Let us make a transformation in which the unknown function $U(\xi)$ is considered as a functional variable of the form

$$U_\xi = F(U) \tag{2.4}$$

and some successive derivatives of U are

$$\begin{aligned} U_{\xi\xi} &= \frac{1}{2}(F^2(U))^{'}, \\ U_{\xi\xi\xi} &= \frac{1}{2}(F^2(U))^{''}\sqrt{F^2(U)}, \\ U_{\xi\xi\xi\xi} &= \frac{1}{2}\left[(F^2(U))^{'''}F^2 + \frac{1}{2}(F^2(U))^{''}(F^2(U))^{'}\right] \end{aligned} \tag{2.5}$$

and so on, where $' = d/dU$.

Step 3. Substituting Eqs.(2.4) and (2.5) into (2.3), we obtain the following ODE

$$R(U, F, F^{'}, F^{''}, F^{'''}, ...) = 0. \tag{2.6}$$

After integration, Equation (2.6) provides the expression of F, and this in turn together with Equation (2.4) gives the appropriate solutions of Equation (2.1). In order to illustrate how the proposed method works, we examine some examples treated by other methods.

3. Applications

3.1. Radhakrishnan-Kundu-Lakshmanan (RKL) equation

In this subsection, we employ the functional variable method(FVM) to obtain new and more general exact solutions of the Radhakrishnan-Kundu-Lakshmanan (RKL) equation, which has been proposed as follows

$$\iota p_t + \alpha p_{xx} + \beta|p|^2 p = \iota\lambda\left(|p|^2 p\right)_x - \iota\delta p_{xxx}, \tag{3.1}$$

where $p = p(x, t)$ is a dependent variable that represents the complex valued wave function with two independent variables of x and t that represent space and time, respectively. The coefficients α and β represent the group-velocity dispersion (GVD) and the nonlinearity terms, respectively. Moreover, λ represents the coefficient of self-steepening for short pulses and δ represents the third-order dispersion (TOD) term.

Assume that the solution of Eq. (3.1) has the form:

$$p(x,t) = U(\zeta)e^{\iota\phi(x,t)}, \tag{3.2}$$

$$\zeta = x - ct, \tag{3.3}$$

$$\phi(x,t) = -\kappa x + \omega t + \theta, \tag{3.4}$$

where $U(\zeta)$ is the shape of the soliton pulse, c is the velocity, $\phi(x,t)$ represents the phase-component, θ is defined as the phase-constant, κ denotes the frequency, and ω is the parameter of wave number. Substituting Eq. (3.2) into Eq. (3.1) and after separating real and imaginary parts, we obtain the following expressions

$$(\alpha + 3\delta\kappa)U_{\zeta\zeta} - (\omega + \alpha\kappa^2 + \delta\kappa^3)U + (\beta - \lambda\kappa)U^3 = 0, \tag{3.5}$$

$$\delta U_{\zeta\zeta\zeta} - (c + 2\alpha\kappa + 3\kappa^2\delta)U_\zeta - 3\lambda U^2 U_\zeta = 0. \tag{3.6}$$

Put $\alpha = -3\delta\kappa, \beta = \lambda\kappa, \omega = 2\delta\kappa^3$ in Eq. (3.5). Then Eq. (3.6) can be reduces to:

$$\delta U_{\zeta\zeta\zeta} - (c - 3\kappa^2\delta)U_\zeta - 3\lambda U^2 U_\zeta = 0. \tag{3.7}$$

Integrating Eq. (3.7) with respect to ζ with zero constants of integration, we obtain

$$\delta U_{\zeta\zeta} - (c - 3\kappa^2\delta)U - \lambda U^3 = 0. \tag{3.8}$$

Substituting Eq. (2.5) into Eq. (3.8), we get

$$\frac{\delta}{2}(F^2(U))' = \lambda U^3 + (c - 3\kappa^2\delta)U. \tag{3.9}$$

Integrating Eq. (3.9) with respect to U with zero constants of integration, we have

$$F(U) = \pm\sqrt{\frac{\lambda}{2\delta}}U\sqrt{\left[U^2 + \frac{2(c-3\kappa^2\delta)}{\lambda}\right]}. \tag{3.10}$$

From Eqs. (2.4) and (3.10) we deduce that

$$\int \frac{dU}{U\sqrt{\left[U^2 + \frac{2(c-3\kappa^2\delta)}{\lambda}\right]}} = \pm\sqrt{\frac{\lambda}{2\delta}}(\zeta + \zeta_0) \tag{3.11}$$

where ζ_0 is a constant of integration. After integrating Eq. (3.11), we have the following exact solutions, for $\frac{\lambda}{\delta} > 0$:

$$p_1(x,t) = \sqrt{\frac{2(c-3\kappa^2\delta)}{\lambda}} sech \left[\sqrt{\frac{2(c-3\kappa^2\delta)}{\lambda}} \sqrt{\frac{\lambda}{2\delta}} (x - ct + \zeta_0) \right] e^{\iota(-\kappa x + \omega t + \theta)}, \tag{3.12}$$

$$p_2(x,t) = -\sqrt{\frac{2(c-3\kappa^2\delta)}{\lambda}} csch \left[\sqrt{\frac{2(c-3\kappa^2\delta)}{\lambda}} \sqrt{\frac{\lambda}{2\delta}} (x - ct + \zeta_0) \right] e^{\iota(-\kappa x + \omega t + \theta)}. \tag{3.13}$$

For $\frac{\lambda}{\delta} < 0$, we obtain periodic solutions as follows:

$$p_3(x,t) = \sqrt{\frac{2(c-3\kappa^2\delta)}{\lambda}} sec \left[\sqrt{\frac{2(c-3\kappa^2\delta)}{\lambda}} \sqrt{\frac{\lambda}{2\delta}} (x - ct + \zeta_0) \right] e^{\iota(-\kappa x + \omega t + \theta)}, \tag{3.14}$$

$$p_4(x,t) = \sqrt{\frac{2(c-3\kappa^2\delta)}{\lambda}} csc \left[\sqrt{\frac{2(c-3\kappa^2\delta)}{\lambda}} \sqrt{\frac{\lambda}{2\delta}} (x - ct + \zeta_0) \right] e^{\iota(-\kappa x + \omega t + \theta)}. \tag{3.15}$$

3.2. Landau-Ginzburg-Higgs equation

The Landau-Ginzburg-Higgs equation is stated as

$$v_{tt} - v_{xx} - g^2 v + h^2 v^3 = 0, \tag{3.16}$$

where $q(x,t)$ symbolizes the electrostatic potential of the ion-cyclotron wave x and t stand for the nonlinearized spatial and temporal coordinates and g and h are real parameters. The NLEE (3.16) was formulated by Lev Devidovich Landau and Vitaly Lazarevich Ginzburg with broad applications for the explanation of superconductivity and drift cyclotron waves in radially inhomogeneous plasma for coherent ion-cyclotron waves.

A significant NLEE is the Landau-Ginzburg-Higgs equation, which deals with the internal processes of complex physical phenomena that explain superconductivity and drift cyclotron waves in radially inhomogeneous plasma for coherent ion-cyclotron waves. To search compatible solitary solutions, we will now use the functional variable method. Introducing the wave variable

$$v(x,t) = U(\zeta), \zeta = \omega x - ct, \tag{3.17}$$

where c refers to the velocity of soliton and ω implies the wave number, substituting (3.17) into Eq. (3.16), we accomplish an ODE of the structure

$$(c^2 - \omega^2) U_{\zeta\zeta} - g^2 U + h^2 U^3 = 0. \tag{3.18}$$

Substituting Eq. (2.5) into Eq. (3.18), we get

$$\frac{(c^2-\omega^2)}{2}\left(F^2(U)\right)' = g^2U - h^2U^3. \tag{3.19}$$

Integrating Eq. (3.19) with respect to U with zero constants of integration, we have

$$F(U) = \pm\frac{g}{\sqrt{(c^2-\omega^2)}}U\sqrt{\left[1-\frac{h^2}{2g^2}U^2\right]}. \tag{3.20}$$

From Eqs. (2.4) and (3.20) we deduce that

$$\int\frac{dU}{U\sqrt{\left[1-\frac{h^2}{2g^2}U^2\right]}} = \pm\frac{g}{\sqrt{(c^2-\omega^2)}}(\zeta+\zeta_0), \tag{3.21}$$

where ζ_0 is a constant of integration. After integrating Eq. (3.21), we have the following exact solutions, for $\frac{g^2}{c^2-\omega^2} > 0$:

$$v_1(x,t) = \frac{\sqrt{2}g}{h}sech\left[\frac{g}{\sqrt{(c^2-\omega^2)}}(\omega x - ct)+\zeta_0\right], \tag{3.22}$$

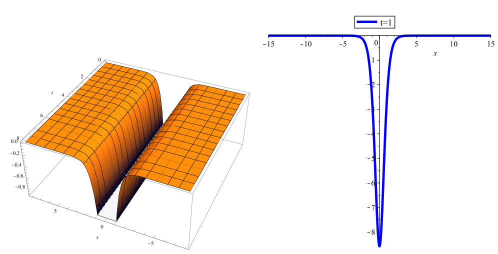
$$v_2(x,t) = -\frac{\sqrt{2}g}{h}csch\left[\frac{g}{\sqrt{(c^2-\omega^2)}}(\omega x - ct)+\zeta_0\right]. \tag{3.23}$$

For $\frac{g^2}{c^2-\omega^2} < 0$, we obtain periodic solutions as follows:

$$v_3(x,t) = \frac{\sqrt{2}g}{h}sec\left[\frac{g}{\sqrt{(c^2-\omega^2)}}(\omega x - ct)+\zeta_0\right], \tag{3.24}$$

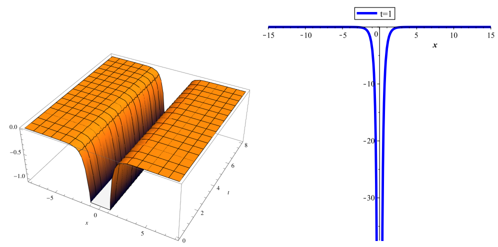
$$v_4(x,t) = \frac{\sqrt{2}g}{h}csc\left[\frac{g}{\sqrt{(c^2-\omega^2)}}(\omega x - ct)+\zeta_0\right]. \tag{3.25}$$

4. Graphical representation of the RKL and Landau-Ginzburg-Higgs equation



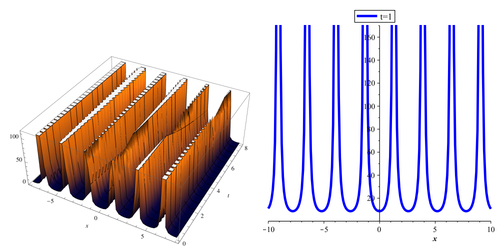
(a) (b)

Figure 1. (a) 3D (b) 2D graphs of $|p_1(x,t)|^2$ given in Eq. (3.12) with $c = -0.01, \delta = -2, \kappa = 0.7, \lambda = -2, \zeta_0 = 0$.



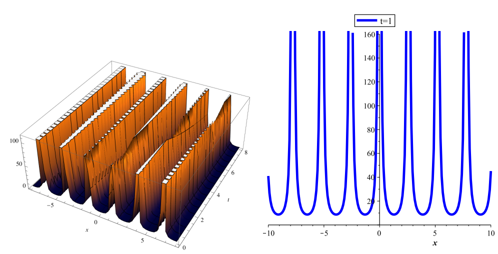
(a) (b)

Figure 2. (a) 3D (b) 2D graphs of $|p_2(x,t)|^2$ given in Eq. (3.13) with $c = -0.01, \delta = -2, \kappa = 0.7, \lambda = -2, \zeta_0 = 0$.



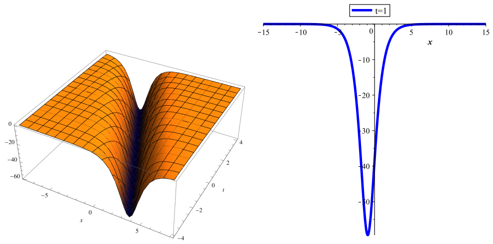
(a) (b)

Figure 3. (a) 3D (b) 2D graphs of $|p_3(x,t)|^2$ given in Eq. (3.14) with $c = -0.01, \delta = -2, \kappa = 0.7, \lambda = -2, \zeta_0 = 0$.



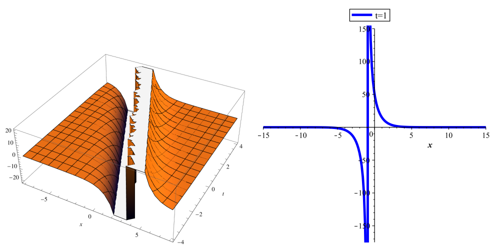
(a) (b)

Figure 4. (a) 3D (b) 2D graphs of $|p_4(x,t)|^2$ given in Eq. (3.15) with $c = -0.01, \delta = -2, \kappa = 0.7, \lambda = -2, \zeta_0 = 0$.



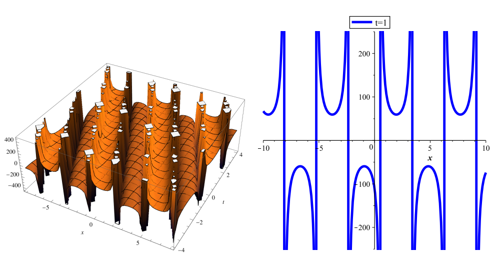
(a) (b)

Figure 5. (a) 3D (b) 2D graphs of $v_1(x,t)$ given in Eq. (3.22) with $c = -0.12, g = -0.42, h = 0.01, \omega = 0.13, \zeta_0 = 0$.



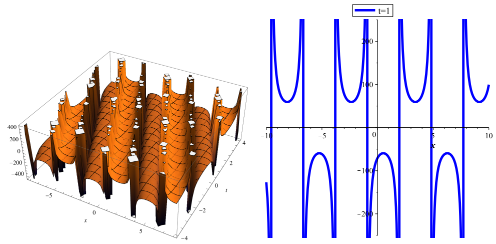
(a) (b)

Figure 6. (a) 3D (b) 2D graphs of $v_2(x,t)$ given in Eq. (3.23) with $c = -0.12, g = -0.42, h = 0.01, \omega = 0.13, \zeta_0 = 0$.



(a) (b)

Figure 7. (a) 3D (b) 2D graphs of $v_3(x,t)$ given in Eq. (3.24) with $c = -0.12, g = -0.42, h = 0.01, \omega = 0.13, \zeta_0 = 0$.



(a) (b)

Figure 8. (a) 3D (b) 2D graphs of $v_4(x,t)$ given in Eq. (3.25) with $c = -0.12, g = -0.42, h = 0.01, \omega = 0.13, \zeta_0 = 0$.

We have presented graphs of solitary waves for equations (3.12)-(3.15) and (3.22)-(3.25), generated by selecting appropriate values for the unknown parameters involved [45], to better understand the underlying mechanisms of the original physical phenomena. Graphical representation serves as an effective communication tool, clearly illustrating the solutions to the problems. These graphical depictions are shown in Fig.1 to Fig.8. Solitary and periodic wave solutions are significant types of solutions for nonlinear partial differential equations, as many such equations exhibit a variety of solitary wave solutions. Solitons, a specific class of solutions to nonlinear partial differential equations with weak linearity, are often used to describe physical systems. The existence of periodic traveling waves typically depends on the parameter values in the mathematical equations, with parameters influencing both amplitude and velocity. A soliton is a self-sustaining wave packet that preserves its shape while traveling at a constant velocity.

5. Conclusions

The functional variable method was effectively employed to derive precise traveling wave solutions for the Radhakrishnan-Kundu-Lakshmanan (RKL) and Landau-Ginzburg-Higgs equations. This approach is simpler and faster compared to conventional methods. Additionally, it is straightforward, concise, and particularly well-suited for computer implementation. The algebraic complexities and lengthy calculations were efficiently handled using the symbolic computation software Mathematica. Consequently, this method can be expanded to address nonlinear problems encountered in soliton theory and other related fields.

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