A Mathematical Modeling of Two-Strain Tuberculosis Transmission: Deterministic and Stochastic Approaches to Screening, Treatment, and Quarantine Strategies*

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Abstract This research observes the transmission dynamics of drug-sensitive tuberculosis (DS-TB), drug-resistant tuberculosis (DR-TB) and undiagnosed tuberculosis infections using deterministic and stochastic models. The study focuses on the impacts of contact rates, screening of latent individuals or high-risk groups, treatment, and quarantine measures on the basic reproduction rate (R_0) and the estimated number of infected individuals (m). The results show that undiagnosed infected individuals are the greatest factor on the spread of tuberculosis within the system, which emphasizes the importance of screening on latent individuals. Analysis on the effects of screening on latent individuals also emphasizes the importance of treatment and quarantine for both DS-TB and DR-TB infections, respectively. However, analysis on the effects of both treatment and quarantine states that relying on either treatment or quarantine efforts is not enough to stop the spread of the two-strain TB within the system; thus a combined strategies is required to help eradicate the disease.

Keywords Deterministic model, stochastic model, screening, treatment, quarantine

MSC(2010) 60G05, 60G07, 60J27, 60J28, 60J80, 65C40.

1. Introduction

Tuberculosis (TB) is one of the long-standing health problems faced by the global community. Despite being a preventable and curable disease, TB remains a major global health threat, causing significant morbidity and mortality around the world [1]. World Health Organization (WHO) reported in the Global Tuberculosis Report in 2022 that there were an estimated 969,000 tuberculosis (TB) cases in Indonesia.

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This makes Indonesia the second largest contributor to TB cases in the world after India [2].

Tuberculosis (TB) spreads from person to person through airborne particles containing *Mycobacterium tuberculosis* (MTB). This means that TB can be transmitted through the air when an infected person coughs, sneezes, or talks, releasing droplets that contain the bacteria. The bacteria can then be inhaled by a healthy person, leading to infection [3,4]. Once infected, TB bacteria can reside in the body without causing noticeable clinical symptoms, known as latent TB infection. However, the latent MTB bacteria can become active, which may take weeks, months, or even the entire lifetime of the infected person [4].

Drug-resistant TB is a significant threat to TB control globally, requiring effective treatment strategies to improve treatment outcomes and reduce mortality [5]. Drug-resistant TB (DR-TB), which is resistant to at least rifampicin and isoniazid, the two most potent first-line anti-TB drugs, is a particularly concerning form of drug-resistant and difficult-to-treat TB [5,6]. New prevention and treatment guideline options are being developed to address this challenge [5].

Mathematical modeling of tuberculosis (TB) transmission plays a crucial role in understanding the dynamics of the disease, offering valuable insights into its epidemiology and informing the development of effective control strategies. Over the past few decades, a diverse group of mathematicians, statisticians, and biologists have contributed to the establishment of various transmission dynamic models for TB [7–16].

By analyzing the dynamics of TB transmission and the impact of different interventions, mathematical models can help identify the most effective ways to prevent and treat TB and the optimal strategies for controlling the disease [6,7]. For instance, Jinhui Zhang et al. [8] developed a mathematical model of tuberculosis (TB) transmission in China. The model is based on the data of TB incidence and mortality rates in China from 1990 to 2015. The article concluded that the mathematical model of TB transmission in China is a useful tool for understanding the dynamics of TB transmission and for evaluating the effectiveness of different control strategies. The model suggested that the transmission dynamics of TB in China are influenced by a variety of factors, including demographic and economic factors. The model also suggested that the control of TB in China will require a combination of treatment and control measures, as well as changes in demographic and economic factors [8].

Ayinla et al. [9] developed a mathematical model for TB that employs a compartmental approach, incorporating seven distinct classes: susceptible (S), vaccinated (V), exposed (E), undiagnosed infectious (I_1), diagnosed infectious (I_2), treated (T), and recovered (R). The model was designed to analyze the dynamics of TB transmission and evaluate the impact of various control strategies, such as vaccination, diagnosis, and treatment. Their stability analysis revealed that due to the occurrence of backward bifurcation, merely reducing the basic reproduction number below one ($R_0 < 1$) is insufficient to eradicate TB. The study underscored the necessity of high vaccination rates, effective diagnostic methods, and timely treatment to manage and control the TB epidemic. Prioritizing diagnosis over treatment is particularly emphasized, as early detection is crucial for effective disease management [9].

Kasbawati et al. [10] developed a deterministic model for TB infections and compared it with a continuous Markov chain model. The study aimed to analyze