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Stability Analysis of a Diffusive Ratio-Dependent Predator-Prey Model with Nonlocal Perception*

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Abstract In this paper, we propose a diffusive ratio-dependent predator-prey model with a nonlocal perceptual term. We first discuss the well-posedness of solutions. Then for the model without the nonlocal perception term, we obtain stability conditions of the spatially homogeneous steady state. Under these conditions, when considering the effects of nonlocal perception, our result indicates that if the system without nonlocal perception term is stable, and even after introducing nonlocal perception term, the system remains stable. However, if the system without nonlocal perception term is unstable, and the introduction of nonlocal perception term will make the system become stable.

Keywords Predator-prey model, nonlocal perception, stability

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1. Introduction

Reaction-diffusion systems are crucial for understanding various phenomena in physics, chemistry, biology, and ecology. These systems typically describe the temporal and spatial evolution of concentrations of chemicals, biological populations, or other entities through reactions (interactions between species) and diffusion (propagation in space). It's known that spatial dispersion is a key factor contributing to the spatial heterogeneity that leads to the formation of spatial patterns. The significance of spatial models has been acknowledged by biologists for many years and has become a central topic in both ecology and mathematical ecology, see, e.g., [4, 17, 18].

A classic example is the Lotka-Volterra model, which has been extensively studied in the context of predator-prey dynamics. A traditional predator-prey model, which does not take into account spatial effects, can be described by the following system:

$$\begin{cases} \frac{dN}{dt} = rN(1 - \frac{N}{K}) - G(N,P)P, \\ \frac{dP}{dt} = \eta G(N,P)P - \gamma P, \end{cases} \tag{1.1}$$

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where N and P represent the population densities of prey and predators, respectively. The parameter $r > 0$ denotes the intrinsic growth rate of the prey, while K symbolizes the environmental carrying capacity. The function $G(N, P)$ means predation, referred to as the functional response. The parameter η represents the efficiency of converting biomass from predation, and γ stands for the per-capita death rate of the predators.

There are many works dedicated to studying (1.1), which indicates that ratio-dependent predation can support very rich dynamics. For example, when $G(N, P) = \frac{\alpha N}{N+aP}$, Chen et al. [9] explored complex dynamics in a ratio-dependent predator-prey model that includes the Allee effect and predator harvesting. When $G(N, P) = \frac{bN}{1+aN}$, Haque [12] showed complexity of the emergence of spatiotemporal within ratio-dependent predator-prey systems, focusing on patterns such as Turing patterns and wave bifurcations. And he concluded how spatial factors and predator-prey ratios can influence the overall dynamics of ecosystems. Lan et al. [14] investigated the dynamics of a ratio-dependent predator-prey model under the influence of stochastic environments, incorporating Holling III functional response and adding nonlinear harvesting terms. Tyutyunov and Titova [23] studied the dynamics of ratio-dependent predator-prey models with free boundaries, focusing on the long-term behaviors of both predator and prey populations. Xiao and Ruan [25] provided a comprehensive analysis of the global dynamics associated with ratio-dependent models. They addressed issues such as stability, bifurcations, and the conditions that lead to extinction or coexistence of species.

Arditi and Ginzburg [3] suggested the following ratio-dependent prey-predator model:

$$\begin{cases} \frac{dN}{dt} = rN(1 - \frac{N}{K}) - \frac{\alpha NP}{P+\alpha\beta N}, \\ \frac{dP}{dt} = \frac{\eta\alpha NP}{P+\alpha\beta N} - \gamma P, \end{cases} \tag{1.2}$$

where α and β represent the predator's attack rate and handing time, respectively. The biological meanings of other parameters in the model are the same as those in (1.1). Introducing the diffusion terms into system (1.2), the following ratio-dependent reaction-diffusion system is obtained:

$$\begin{cases} \frac{\partial N}{\partial t} = d_1 N_{xx} + rN(1 - \frac{N}{K}) - \frac{\alpha NP}{P+\alpha\beta N}, \\ \frac{\partial P}{\partial t} = d_2 P_{xx} + \frac{\eta\alpha NP}{P+\alpha\beta N} - \gamma P, \end{cases} \tag{1.3}$$

where d_1 and d_2 are the diffusion rates of the prey and predator.

By introducing the following transformations,

$$u = \frac{\alpha\beta}{\eta K}N, v = \frac{\alpha\beta}{\eta^2 K}P, \tilde{t} = \frac{\eta}{\beta}t, \hat{x} = \sqrt{\frac{\eta}{\beta}}x,$$

system (1.3) is simplified as

$$\begin{cases} \frac{\partial u}{\partial t} = d_1 u_{xx} + au(1 - \frac{u}{b}) - \frac{buv}{bu+v}, \\ \frac{\partial v}{\partial t} = d_2 v_{xx} - cv + \frac{buv}{bu+v}, \end{cases} \tag{1.4}$$

where the other nondimensional parameters are

$$a = \frac{r\beta}{\eta}, b = \frac{\alpha\beta}{\eta}, c = \frac{\gamma\beta}{\eta}.$$

And for simplicity of notations, the over-bars has been removed from the model (1.4). The model parameters d_1, d_2, a, b and c are all positive constants.

The dynamics of diffusive system (1.4) and some relevant systems have been recently studied in [6, 8, 15, 22]. Bartumeus et al. [6] studied the dynamic behavior of a ratio-dependent predator-prey model in a reaction-diffusion system to explore the conditions for diffusion-driven instabilities (i.e., Turing structures) by using linear stability analysis. They concluded that predator density negatively affects the prey consumption rate of an average predator due to mutual interference among predators. Chen et al. [8] investigated a delayed reaction-diffusion predator-prey model that incorporates fear and anti-predator behaviors and studied stability, bifurcation, and pattern formation under these additional biological factors. Lazaar et al. [15] studied the stability of a predator-prey model that includes both diffusion effects and prey refuge. Their analysis covers global stability and the impact of incorporating prey refuge into the dynamics of the system. Song and Zou [22] focused on system (1.4). They investigated the stability of positive constant equilibrium, identified conditions under which Turing instability occurs, and studied the existence of Hopf and steady state bifurcations.

Reaction-diffusion system with nonlocal interactions has been studied extensively over the past decades, mainly due to the importance of these models in understanding complex biological interactions. To the best of our knowledge, there are two ways to introduce nonlocal effects into a system. The first approach is to incorporate nonlocal effects into the response function. From a biological perspective, this means that predators and prey interact over a spatial range rather than just at a single point. Considering the situation where individuals compete for common and rapidly balanced resources, Furter and Grinfeld [11] examined two models of single-species dynamics which incorporate nonlocal effects. They focused on the ability of these models to generate stable patterns and investigated the global behavior of the bifurcating branches. Banerjee et al. [5] investigated a predator-prey model incorporating nonlocal consumption, focusing on the bistable dynamics of prey and its impact on the formation of spatial patterns. Bayliss and Volpert [7] examined competitive population models with species-specific nonlocal coupling, providing insights into how the nonlocal term can be integrated into predator-prey models, where the nonlocal term is expressed in integral form to represent resource competition. Readers can also refer to references [19, 20, 26] for other related research works.

Another way of introducing nonlocal effects is attributed to the work of Fagan et al. [10]. They proposed a nonlocal advection-diffusion equation, which incorporates nonlocal perception into the diffusion term of the equation. And they wanted to explore how foragers can utilize nonlocal information to improve their foraging success in dynamic environments. Their research suggests that nonlocal information can be highly beneficial, increasing the spatiotemporal concentration of foragers on resources by up to twofold compared to movement based on purely local information. However, their model ignored the birth/death process of the foragers and the mechanistic dynamics of resource availability. Recently, Song et al. [21] incorporated these important missing components and proposed a consumer-resource model with nonlocal perception. The nonlocal perception term is expressed as $\alpha(vh_x)_x$, where v denotes the density of foragers, $\alpha > 0$ means the perception strength and $h(x,t)$ defines the forager's resource perception function.

Inspired by the above work, especially references [10], [21] and [22], we propose the following model by introducing a nonlocal perceptual term into the system (1.4) and supplementing it with Neumann boundary conditions and nonnegative initial conditions.

$$\begin{cases} \frac{\partial u}{\partial t} = d_1 u_{xx} + au(1-\frac{u}{b}) - \frac{buv}{bu+v}, & x \in \Omega, t > 0, \\ \frac{\partial v}{\partial t} = d_2 v_{xx} - \alpha(v h_x)_x + (\frac{bu}{bu+v} - c)v, & x \in \Omega, t > 0, \\ \frac{\partial u(x,t)}{\partial \nu} = \frac{\partial v(x,t)}{\partial \nu} = 0, & x \in \partial\Omega, t \geq 0, \\ u(x,0) = u_0(x) \geq 0,\ v(x,0) = v_0(x) \geq 0, x \in \Omega, \end{cases} \tag{1.5}$$

where the perception function $h(x,t)$ of predators towards prey is defined as

$$h(x,t) = \int_\Omega k(x-y)u(y,t)dy,$$

and $k(x-y)$ is some reasonable kernel function (also known as the detection function), which can be taken different functions according to different regions. The reasonable kernel function $k(x)$ should satisfy the following properties [24]:
(i) $k(x)$ is symmetric about the origin;
(ii)$\int_\Omega k(x)dx = 1$;
(iii)$\lim_{r\to 0^+} k(x) = \delta(x)$;
(iv) $k(x)$ is nonincreasing from the origin.
Additionally, in this paper, for the biological relevance, we also assume that the detection function $k(x-y)$ should satisfy the following conditions:

$$k(x-y) > 0,\ k(x-y) = k(y-x),\ \int_\Omega k(x-y)dy = 1, \quad x, y \in \Omega.$$

The purpose of this paper is to analyze the dynamic behavior of system (1.5). Especially, we focus on the impact of nonlocal perceptual coefficient α on the dynamics of system (1.5). For the sake of simplicity in the discussion, we consider the system in one-dimensional space, i.e., throughout this paper, we always assume that $\Omega = (0, l\pi)$ with $l > 0$.

The remainder of this article is arranged as follows. In Section 2, we discuss the well-posedness of solutions for (1.5) by applying some basic theories and methods such as the upper and lower solution methods and comparison principles of parabolic system. By applying qualitative theory of differential equations and the implicit function theorem, the stability of positive constant steady state of system (1.4) and system (1.5) in a bounded region $(0, l\pi)$ is investigated respectively in Section 3. Finally, we conclude this paper with a short discussion in Section 4.

Throughout this paper, $\mathbb{N}$ represents the set of all positive integers. $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ represents the set of all nonegative integers. $\mathbb{R}$ represents the set of all real numbers.

2. Well-posedness of solutions

In this section, we mainly discuss the well-posedness of the initial-boundary value problem for system (1.5). Here we choose

$$X = C(\bar{\Omega}; \mathbb{R}), \quad \bar{\Omega} = [0, l\pi]$$

and the realization $A : D(A) \longmapsto X$ of the operator $\mathcal{A}$ in X, with domain

$$D(A) = \left\{ u, v \in W^{2,p}(\bar{\Omega}), \forall p > 1 : \mathcal{A}u, \mathcal{A}v \in X, \frac{\partial u(x,t)}{\partial \nu}\Big|_{\partial\Omega} = \frac{\partial v(x,t)}{\partial \nu}\Big|_{\partial\Omega} = 0 \right\},$$

where $\mathcal{A} = \frac{\partial^2}{\partial x^2}$. Let

$$f(u,v) = au(1 - \frac{u}{b}) - \frac{buv}{bu + v}, \ g(u,v) = (\frac{bu}{bu+v} - c)v.$$

Then it is easy to verify that f and g satisfy the following conditions:
(f_1) The function $f \in C^1([0,\infty] \times (0,\infty], \mathbb{R})$, $f(0,v) = 0$ for $v > 0$, and there exists a function $F(u) = au(1 - \frac{u}{b}) : [0,\infty) \to \mathbb{R}$ and a positive constant b such that $f(u,v) < F(u)$, and $F(u)$ satisfies $F(0) = 0$, $F(u) < 0$ when $u > b$.
(g_1) The function $g \in C^1((0,\infty] \times [0,\infty], \mathbb{R})$, $g(u,0) = 0$ for $u > 0$, and there exist $K_1 = 1 + c > 0$, $K_2 = c > 0$ such that $g(u,v) \leq (K_1 + K_2 u)v$.

Theorem 2.1. *Let d_1, d_2, a, b be positive constants, $\alpha \geq 0$, and $0 < c \leq 1$. Assume that f and g satisfy conditions (f_1) and (g_1), respectively. Then, for $\Omega = (0, \ell\pi)$, system (1.5) with the initial condition $u_0 \in C^2(\bar{\Omega}), v_0 \in C^2(\bar{\Omega})$ possesses a unique solution (u(x, t), v(x, t)) for (x, t)$\in \bar{\Omega} \times [0,\infty)$, and $u, v \in C^{2,1}(\bar{\Omega} \times [0,\infty))$ if the detection function k satisfies*

$$(H_1) : \int_\Omega \left(\int_\Omega \partial_x k(x-y) dy \right)^p dx < +\infty, for\ all\ p > 1.$$

Moreover, if $u_0(x) > 0$ for $x \in \bar{\Omega}$, $v_0(x) \geq (\not\equiv) 0$ for $x \in \bar{\Omega}$, then $u(x,t) > 0, v(x,t) > 0$ for $(x,t) \in \bar{\Omega} \times [0,\infty)$.

Proof. For the sake of clarity, we will prove this theorem by three steps.
Step 1. We first prove that there exists $\delta > 0$ such that system (1.5) has a unique solution (u, v) in $C^{2,1}(\bar{\Omega} \times [0,\delta])$.

Define

$$h_0(x) = \int_0^{l\pi} k(x-y) u_0(y) dy,$$
$$F^{(1)}(t,x,u,v) = au(x,t)(1 - \tfrac{u(x,t)}{b}) - \tfrac{bu(x,t)v(x,t)}{bu(x,t)+v(x,t)},$$
$$F^{(2)}(t,x,u,v,v_x) = \alpha \left(v(x,t) \left(h_0 \right)_x \right)_x + \left(\tfrac{bu(x,t)}{bu(x,t)+v(x,t)} - c \right) v(x,t).$$

It is noted that we choose the Green function of the operator $-d_3 \frac{\partial^2}{\partial x^2} + I$ with Neumann boundary condition as the detection function, i.e., $k(x-y)$ is the solution of the following problem

$$\begin{cases} -d_3 \frac{\partial^2 k}{\partial x^2} + k = \delta(x-y), \ x \in (0, l\pi), \\ k_x(0-y) = k_x(l\pi - y) = 0. \end{cases} \tag{2.1}$$

Then from the expression of $h_0(x)$, the properties of $k(x-y)$ and the initial conditions, it is easy to prove that $(h_0)_x$, $(h_0)_{xx}$ is bounded, i.e., there exist positive constants A and B such that

$$|(h_0)_x| \leq A, \ |(h_0)_{xx}| \leq B.$$

From the function space where u, v are located, we know that for any $(x,t) \in [0, l\pi] \times [0, T]$, there exist positive constants M and N such that

$$|u(x,t)| \leq M, |v(x,t)| \leq N.$$

For any $0 \leq s < t \leq T$ and $\theta \in (0,1)$, by calculation, we have

$$\begin{aligned}
&\left|F^{(1)}(t,x,u_1(x,t),v_1(x,t)) - F^{(1)}(s,x,u_2(x,s),v_2(x,s))\right| \\
=&\Bigg|\left[au_1(x,t)\left(1-\frac{u_1(x,t)}{b}\right) - \frac{bu_1(x,t)v_1(x,t)}{bu_1(x,t)+v_1(x,t)}\right] \\
&-\left[au_2(x,s)\left(1-\frac{u_2(x,s)}{b}\right) - \frac{bu_2(x,s)v_2(x,s)}{bu_2(x,s)+v_2(x,s)}\right]\Bigg| \\
\leq&\left|au_1(x,t)\left(1-\frac{u_1(x,t)}{b}\right) - au_2(x,s)\left(1-\frac{u_2(x,s)}{b}\right)\right| \\
&+\left|\frac{bu_1(x,t)v_1(x,t)}{bu_1(x,t)+v_1(x,t)} - \frac{bu_2(x,s)v_2(x,s)}{bu_2(x,s)+v_2(x,s)}\right| \\
\leq&\left|a-\frac{a}{b}(u_1+u_2)\right||u_1-u_2| + \frac{b^2u_1u_2}{(bu_1+v_1)(bu_2+v_2)}|v_1-v_2| \\
&+\frac{bv_1v_2}{(bu_1+v_1)(bu_2+v_2)}|u_1-u_2| \\
\leq&\left|a+\frac{a}{b}(u_1+u_2)\right|\|u_1-u_2\|_{C^1} + \|v_1-v_2\|_{C^1} + b\|u_1-u_2\|_{C^1} \\
\leq&L_1\left(\|u_1-u_2\|_{C^1} + \|v_1-v_2\|_{C^1} + |t-s|^\theta\right),
\end{aligned} \tag{2.2}$$

and

$$\begin{aligned}
&\left|F^{(2)}(t,x,u_1(x,t),v_1(x,t),(v_1)_x(x,t)) - F^{(2)}(s,x,u_2(x,s),v_2(x,s),(v_2)_x(x,s))\right| \\
=&\Bigg|\left[\alpha\big(v_1(x,t)(h_0)_x\big)_x + \frac{bu_1(x,t)v_1(x,t)}{bu_1(x,t)+v_1(x,t)} - cv_1(x,t)\right] \\
&-\left[\alpha\big(v_2(x,s)(h_0)_x\big)_x + \frac{bu_2(x,s)v_2(x,s)}{bu_2(x,s)+v_2(x,s)} - cv_2(x,s)\right]\Bigg| \\
\leq&\left|\alpha(v_1(x,t)-v_2(x,s))_x(h_0)_x\right| + \left|\alpha(v_1(x,t)-v_2(x,s))(h_0)_{xx}\right| \\
&+\left|\frac{bu_1(x,t)v_1(x,t)}{bu_1(x,t)+v_1(x,t)} - \frac{bu_2(x,s)v_2(x,s)}{bu_2(x,s)+v_2(x,s)}\right| + \left|c(v_2(x,s)-v_1(x,t))\right| \\
\leq&\alpha A\|(v_1)_x-(v_2)_x\|_{C^1} + \alpha B\|v_1-v_2\|_{C^1} + \|v_1-v_2\|_{C^1} + b\|u_1-u_2\|_{C^1} \\
&+c\|v_1-v_2\|_{C^1} \\
\leq&L_2\big(\|u_1-u_2\|_{C^1} + \|v_1-v_2\|_{C^1} + \|(v_1)_x-(v_2)_x\|_{C^1} + |t-s|^\theta\big),
\end{aligned} \tag{2.3}$$

where

$$L_1 = \max\left\{a+b+\frac{2Ma}{b}, 1\right\}, L_2 = \max\left\{\alpha B + 1 + c, b, \alpha A\right\}.$$

Taking $L = \max\{L_1, L_2\}$, from (2.2) and (2.3), we obtain that there exists $\theta \in (0,1)$ such that

$$\left|F^{(1)}(t,x,u_1(x,t),v_1(x,t) - F^{(1)}(s,x,u_2(x,s),v_2(x,s))\right|$$

$$\leq L(\|u_1 - u_2\|_{C^1} + \|v_1 - v_2\|_{C^1} + |t-s|^\theta),$$

and

$$\Big|F^{(2)}(t,x,u_1(x,t),v_1(x,t),(v_1)_x(x,t)) - F^{(2)}(s,x,u_2(x,s),v_2(x,s),(v_2)_x(x,s))\Big|$$
$$\leq L(\|u_1 - u_2\|_{C^1} + \|v_1 - v_2\|_{C^1} + \|(v_1)_x) - (v_2)_x)\|_{C^1} + |t-s|^\theta),$$

which means that $F^{(1)}$ and $F^{(2)}$ satisfy a Hölder condition with respect to t, and a Lipschitz condition with respect to u and v for $F^{(1)}$. Meanwhile, a Lipschitz condition with respect to u, v and v_x for $F^{(2)}$. Combining (H_1) condition, we know that there exists $\delta > 0$ such that $u, v \in C^{2,1}([0, l\pi] \times [0, \delta])$ by applying the proposition 7.3.3 in [16].

Step 2. We will continue to show that both u and v are uniformly bounded on $[0, l\pi] \times [0, \delta]$.

Thanks to condition (f_1), there exist $c_1 = a + 1$ and $c_2 = Ml\pi + 1$ such that $a(1 - \frac{u}{b}) - \frac{bv}{bu+v} \leq a(1 - \frac{u}{b}) \leq a < c_1$ and $\sup_{t\in[0,\delta]} \int_0^{l\pi} u\, dx \leq \sup_{t\in[0,\delta]} \int_0^{l\pi} |u|\, dx \leq Ml\pi < c_2$. According to theorem 3.1 in [2], we can obtain that $u(x,t)$ is uniformly bounded on $[0, l\pi] \times [0, \delta]$, i.e., there is a constant $B_0 > 0$ such that $|u(x,t)| \leq B_0$ for any $(x,t) \in [0, l\pi] \times [0, \delta]$.

It follows from condition (g_1) that $v(x,t)$ satisfies

$$\begin{cases} v_t(x,t) \leq d_2 v_{xx} - \alpha(h_0)_x v_x + (K_1 + K_2 B_0 - \alpha(h_0)_{xx})v, & x \in (0, l\pi),\ 0 < t < \delta, \\ v_x(0,t) = v_x(l\pi, t) = 0, & 0 < t < \delta, \\ v(x,0) = v_0(x), & x \in (0, l\pi). \end{cases}$$

From Theorem 3.1 in [1] and the results in [13], we can also obtain that $v(x,t)$ is uniformly bounded on $[0, l\pi] \times [0, \delta]$, i.e., there is a constant $C_0 > 0$ such that $|v(x,t)| \leq C_0$ for $\forall (x,t) \in [0, l\pi] \times [0, \delta]$.

Step 3. We will verify the positivity of the solution (u, v) on $[0, l\pi] \times [0, \delta]$.

At first, we verify the positivity of the solution $u(x,t)$, which satisfies the following initial-boundary value problem

$$\begin{cases} \frac{\partial u}{\partial t} = d_1 u_{xx} + au(1 - \frac{u}{b}) - \frac{buv}{bu+v}, & x \in (0, l\pi), 0 < t < \delta, \\ u_x(0,t) = u_x(l\pi, t) = 0, & 0 < t < \delta, \\ u(x,0) = u_0(x) \geq 0, & x \in (0, l\pi). \end{cases} \tag{2.4}$$

Treating (2.4) as a scalar equation in u, by using condition (f_1), we find that $\underline{u} = 0$ is a lower solution to (2.4). Then from the maximum principle for parabolic equations, we know that $u(x,t) \geq 0$ holds for $(x,t) \in [0, l\pi] \times [0, \delta]$ with $u(x,0) = u_0(x) \geq 0$. Moreover, the strong maximum principle shows that if $u(x,0) = u_0(x) > 0$, then $u(x,t) > 0$ for $(x,t) \in [0, l\pi] \times [0, \delta]$.

Next, we verify the positivity of the solution $v(x,t)$, which satisfies the following initial-boundary value problem

$$\begin{cases} \frac{\partial v}{\partial t} = d_2 v_{xx} - \alpha(v(h_0)_x)_x + (\frac{bu}{bu+v} - c)v, & x \in (0, l\pi), 0 < t < \delta, \\ v_x(0,t) = v_x(l\pi, t) = 0, & 0 < t < \delta, \\ v(x,0) = v_0(x) \geq 0, & x \in (0, l\pi). \end{cases} \tag{2.5}$$

Similarly, treating (2.5) as a scalar equation in v, by using condition (g_1), it is easy to know that $\underline{v} = 0$ is a lower solution to (2.5). Again from the maximum principle, we immediately obtain positivity on $v(x,t)$.

Finally, by repeating the above proof process, the solution can be extended to $[0, l\pi] \times [\delta, 2\delta]$ and eventually to $[0, l\pi] \times [0, \infty)$. The proof is complete. □

3. Stability analysis

In this section, we investigate the stability of spatially homogeneous steady state of systems (1.5) in the bounded domain $(0, l\pi)$. Obviously, as long as the following condition is satisfied

$$(C_1) \ 0 < c < 1, a > b(1-c),$$

then $E_* = (u_*, v_*)$ is a positive steady state of (1.4), and it is also a positive steady state of (1.5). Here

$$u_* = \frac{b(a - b(1-c))}{a}, v_* = \frac{b(1-c)u_*}{c} = \frac{b^2(1-c)(a-b(1-c))}{ac}.$$

Additionally, the linearized system of (1.4) at E_* is

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} = D_1 \begin{pmatrix} u_{xx} \\ v_{xx} \end{pmatrix} + A \begin{pmatrix} u \\ v \end{pmatrix}, \tag{3.1}$$

where

$$D_1 = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}, A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \tag{3.2}$$

and

$$a_{11} = \frac{\partial f(u_*, v_*)}{\partial u} = b(1-c^2) - a, \quad a_{12} = \frac{\partial f(u_*, v_*)}{\partial v} = -c^2 < 0,$$
$$a_{21} = \frac{\partial g(u_*, v_*)}{\partial u} = b(1-c)^2 > 0, \quad a_{22} = \frac{\partial g(u_*, v_*)}{\partial v} = -c(1-c) < 0.$$

System(1.4) is subject to the following no-flux boundary condition

$$u_x(0,t) = u_x(l\pi, t) = 0. \tag{3.3}$$

Considering system (3.1) with solutions of the following form:

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} a_n \\ b_n \end{pmatrix} e^{\lambda_n t} \cos(\frac{nx}{l}),$$

then the characteristic equation of (3.1) is

$$\lambda^2 - T_n \lambda + D_n = 0, n \in \mathbb{N}_0,$$

where

$$T_n = b(1-c^2) - a - c(1-c) - (d_1 + d_2) \cdot \left(\frac{n}{l}\right)^2, \tag{3.4}$$

and

$$D_n = d_1 d_2 \left(\frac{n}{l}\right)^4 + d_1 c(1-c)\left(\frac{n}{l}\right)^2 - d_2(b(1-c^2)-a)\left(\frac{n}{l}\right)^2 + c(1-c)(a-b(1-c)). \tag{3.5}$$

For system (1.4) with the boundary condition (3.3), we can obtain the following lemma from (3.4) and (3.5).

Lemma 3.1. *Suppose that d_1, d_2, a, b, c are positive constants, and the condition (C_1) holds, we have*

(1) if $a \geq b(1-c^2)$, then the positive steady state E_* of system (1.4) is locally asymptotically stable;

(2) if $b(1-c^2)-c(1-c) < a < b(1-c^2)$, then the positive steady state E_* of system (1.4) is always locally stable for $d_1 > d_1^*$, and it is unstable for $0 < d_1 < d_1^*$, where d_1^* is given by (3.7).

Proof. (1) If $a \geq b(1-c^2)$, it follows from (3.4) and (3.5) that $T_n < 0$ and $D_n > 0$ for any $n \in \mathbb{N}_0$. And thus the conclusion is confirmed.
(2) If $b(1-c^2)-c(1-c) < a < b(1-c^2)$, according to (3.4), then $T_n < 0$ for any $n \in \mathbb{N}_0$. Next, we will analyze D_n. Firstly, we let $D_n = 0$, which is equivalent to

$$d_1 d_2 \left(\frac{n}{l}\right)^4 + d_1 c(1-c)\left(\frac{n}{l}\right)^2 - d_2(b(1-c^2)-a)\left(\frac{n}{l}\right)^2 + D_0 = 0.$$

Thus there is

$$d_1 = \frac{d_2(b(1-c^2)-a)(\frac{n}{l})^2 - D_0}{d_2(\frac{n}{l})^4 + c(1-c)(\frac{n}{l})^2}.$$

Let $(\frac{n}{l})^2 = y$. Then

$$d_1(y) = \frac{d_2(b(1-c^2)-a)y - D_0}{d_2 y^2 + c(1-c)y}, \quad y > 0.$$

The derivative of $d_1(y)$ with respect to y is

$$d_1'(y) = \frac{(2d_2 y + c(1-c)) \cdot D_0 - d_2^2(b(1-c^2)-a)y^2}{(d_2 y^2 + c(1-c)y)^2}, \; y > 0, \tag{3.6}$$

which implies that $d_1'(y_*) = 0$, where

$$y_* = \frac{D_0 + \sqrt{D_0^2 + c(1-c)(b(1-c^2)-a)D_0}}{d_2(b(1-c^2)-a)} > 0.$$

Then from (3.6), we know that $d_1'(y) > 0$ when $y \in (0, y_*)$, and $d_1'(y) < 0$ when $y \in (y_*, \infty)$. So $d_1(y)$ has a maximum $d_1(y_*)$ at point $y_* \in (0, \infty)$. d_1^* is defined by the following expression:

$$d_1^* = d_1\left(\frac{n_*^2}{l^2}\right) = \max_{n \in \mathbb{N}} \frac{d_2(b(1-c^2)-a)(\frac{n}{l})^2 - D_0}{d_2(\frac{n}{l})^4 + c(1-c)(\frac{n}{l})^2}, \tag{3.7}$$

where

$$n_* = \begin{cases} \lfloor \sqrt{y_*} \cdot l \rfloor, & (d_1(\lfloor \sqrt{y_*} \cdot l \rfloor) > d_1(\lfloor \sqrt{y_*} \cdot l \rfloor) + 1), \\ \lfloor \sqrt{y_*} \cdot l \rfloor + 1, & (d_1(\lfloor \sqrt{y_*} \cdot l \rfloor) \leq d_1(\lfloor \sqrt{y_*} \cdot l \rfloor) + 1). \end{cases}$$

Thus Lemma 3.1 is proved. □

Next, we explore the effect of a nonlocal perception on the stability of system (1.5), where the kernel function $k(x-y)$ takes the solution of system (2.1). Furthermore, under the condition of $a < b(1-c^2)$, it is also assumed that the following condition holds

$$(C_2)\ \frac{d_2}{d_1} > \frac{c(1-c)}{b(1-c^2)-a}.$$

The linearised system of (1.5) at E_* is

$$\begin{pmatrix} u_t \\ v_t \end{pmatrix} = D_1 \begin{pmatrix} u_{xx} \\ v_{xx} \end{pmatrix} + D_2 \begin{pmatrix} h_{xx} \\ 0 \end{pmatrix} + A \begin{pmatrix} u \\ v \end{pmatrix}, \tag{3.8}$$

where D_1, A are given by (3.2), and

$$D_2 = \begin{pmatrix} 0 & 0 \\ -\alpha v_* & 0 \end{pmatrix}.$$

Then, the characteristic equation of (3.8) is

$$\lambda^2 - T_n\lambda + J_n(d_1,\alpha) = 0, n \in \mathbb{N},$$

where

$$J_n(d_1,\alpha) = D_n + \alpha \cdot \frac{b^2c(1-c)(a-b(1-c))(\frac{n}{l})^2}{a(1+d_3(\frac{n}{l})^2)},$$

and T_n, D_n are given by (3.4) and (3.5).

Obviously, for any $n \in \mathbb{N}$, $\frac{b^2c(1-c)(a-b(1-c))(\frac{n}{l})^2}{a(1+d_3(\frac{n}{l})^2)} > 0$, which means that if the positive steady state E_* of system (1.4) is stable, the positive steady state E_* of system (1.5) is also locally asymptotically stable when $\alpha > 0$. That is to say, under the stable condition of system (1.4), the introduction of nonlocal perception term has no impact on the stability of system (1.5).

In what follows, for any fixed $d_1 \in (0, d_1^*)$, we will focus on whether introduction of nonlocal perception term can make system become stable.

Let $J_n(d_1,\alpha) = 0$, we have

$$\alpha = \frac{a(1+d_3(\frac{n}{l})^2)}{-b^2c(1-c)(a-b(1-c))(\frac{n}{l})^2} \cdot D_n.$$

Put $(\frac{n}{l})^2 = z$. Then

$$J_n(d_1,\alpha) > 0 \iff \alpha > (1+d_3z)\cdot h(z),$$

where for any $z > 0$,

$$h(z) = \frac{a[d_1d_2z^2 + d_1c(1-c)z - d_2(b(1-c^2)-a)z + D_0]}{-b^2c(1-c)(a-b(1-c))z}. \tag{3.9}$$

Put $\alpha(z) = (1+d_3z)\cdot h(z)$. Then $\alpha(z)$ and $h(z)$ have the same maximum value point when $z > 0$. Thus we will discuss the maximum value point of $h(z)$ when $z > 0$.

Let $h(z) = 0$. Then we have

$$d_1 d_2 z^2 - [d_2(b(1-c^2) - a) - d_1 c(1-c)]z + D_0 = 0, \ z > 0.$$

By calculating, the two real roots of the above equation are

$$z_{1,2} = \frac{[d_2(b(1-c^2) - a) - d_1 c(1-c)] \mp \sqrt{[d_2(b(1-c^2) - a) - d_1 c(1-c)]^2 - 4d_1 d_2 D_0}}{2d_1 d_2}.$$

It follows from condition (C_2) that

$$z_1 \cdot z_2 = \frac{D_0}{d_1 d_2} > 0, z_1 + z_2 = \frac{d_2(b(1-c^2) - a) - d_1 c(1-c)}{d_1 d_2} > 0,$$

which shows that $z_1, z_2 > 0$. Additionally, there exists a positive constant $z_* = \sqrt{\frac{D_0}{d_1 d_2}}$ such that $h'(z_*) = 0$. Thanks to $z_2 > z_1 > 0$ and $z_1 \cdot z_2 = z_*^2$, we obtain $z_1 < z_* < z_2$.

From (3.9) and (3.10), we obtain that the first derivative and the second derivative of $h(z)$ are

$$h'(z) = \frac{a}{-b^2 c(1-c)(a - b(1-c))} \left(d_1 d_2 - \frac{D_0}{z^2} \right) \tag{3.10}$$

and

$$h''(z) = \frac{a}{-b^2 c(1-c)(a - b(1-c))} \frac{2D_0}{z^3}. \tag{3.11}$$

From (3.10) and (3.11), it is easy to show that $h'(z) > 0$ for $z \in (z_1, z_*)$ and $h'(z) < 0$ for $z \in (z_*, z_2)$. And apparently, $h''(z) < 0$ when $z \in (z_1, z_2)$.

By taking $a = 0.2, \ b = 0.46, \ c = 0.6, \ d_1 = 0.02, \ d_2 = 0.2$, Figure 1. below provides an intuitive description for the function $h(z)$.

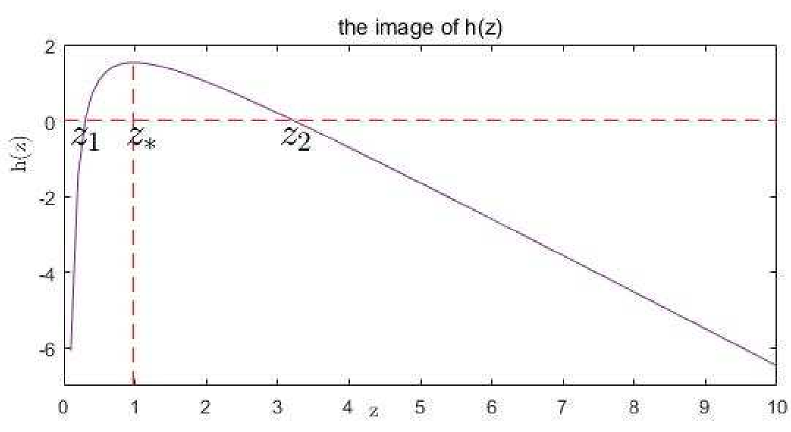

Figure 1. The graph of function h(z) for a=0.2,b=0.46,c=0.6,$d_1 = 0.02, d_2 = 0.2$. z_1, z_2 denote the intersection points of the graph of function $h(z)$ with the z-axis, z_* is the maximum value point of $h(z)$.

In the following we consider the properties of function $\alpha(z)$. It follows from the expression of $\alpha(z)$ that we have

$$\begin{aligned}\alpha(z) &= (1+d_3z)\cdot h(z),\\ \alpha'(z) &= d_3h(z)+(1+d_3z)\cdot h'(z),\\ \alpha'(z_*) &= d_3h(z_*)>0,\\ \alpha'(z_2) &= (1+d_3z)\cdot h'(z_2)<0,\\ \alpha''(z) &= 2d_3h'(z)+(1+d_3z)h''(z).\end{aligned} \tag{3.12}$$

Thanks to $\alpha'(z_*)>0$ and $\alpha'(z_2)<0$, according to the zero-point theorem of closed interval continuous functions, there exists a point $z^*\in(z_*,z_2)$ such that $\alpha'(z^*)=0$. Now let's explain the uniqueness of z^*. In fact, substituting (3.10) and (3.11) into the expression of $\alpha''(z)$ in (3.12), we have

$$\alpha''(z) = -\frac{2a}{b^2c(1-c)(a-b(1-c))}\left(d_1d_2d_3+\frac{D_0}{z^3}\right)<0,$$

which means that $\alpha'(z)$ is a decreasing function when $z>0$. Naturally, $\alpha'(z)$ decreases monotonically in the intervals (z_1,z^*) and (z^*,z_2). Hence we have $\alpha'(z)>\alpha'(z^*)=0$ when $z\in(z_1,z^*)$ and $\alpha'(z)<\alpha'(z^*)=0$ when $z\in(z^*,z_2)$, which shows that z^* is the unique maximum point of $\alpha(z)$ on the interval (z_1,z_2). It is also the only maximum point on the interval $(0,+\infty)$.

Then, we define

$$\alpha_n=\alpha(\frac{n^2}{l^2}),\ \alpha^*=\max_{n\in\mathbb{N}}\ \alpha_n=\max_{n\in\mathbb{N}}\left(\frac{a(1+d_3(\frac{n}{l})^2)}{-b^2c(1-c)(a-b(1-c))(\frac{n}{l})^2}\cdot D_n\right),$$

where

$$n^*=\begin{cases}\lfloor\sqrt{z^*}\cdot l\rfloor, & (\alpha(\lfloor\sqrt{z^*}\cdot l\rfloor)>\alpha(\lfloor\sqrt{z^*}\cdot l\rfloor)+1),\\ \lfloor\sqrt{z^*}\cdot l\rfloor+1, & (\alpha(\lfloor\sqrt{z^*}\cdot l\rfloor)\le\alpha(\lfloor\sqrt{z^*}\cdot l\rfloor)+1).\end{cases}$$

Based on the above discussion, we summarize the following theorem.

Theorem 3.1. *Suppose that d_1,d_2,a,b,c are positive constants, $\alpha\ge0$ and the condition (C_1) holds.*

- *(i) If $a\ge b(1-c^2)$ or $b(1-c^2)-c(1-c)<a<b(1-c^2)$ and $d_1>d_1^*$, then the positive steady state E_* of system (1.5) is locally asymptotically stable. That is to say, under these conditions, the introduction of nonlocal perceptual terms has no impact on stability;*
- *(ii) If $b(1-c^2)-c(1-c)<a<b(1-c^2)$, $0<d_1<d_1^*$ and the condition (C_2) are satisfied, then the positive steady state E_* of system (1.5) is locally asymptotically stable when $\alpha>\alpha^*$.*

4. Conclusion

In this paper, we first discuss the well-posedness of solutions for system (1.5) with nonlocal perception term. For system (1.4) without nonlocal perception term, we obtain the conditions for the stability and instability of positive steady state E_* in one dimensional space $\Omega=(0,l\pi)$. The results show that if the system without nonlocal perception term is stable, then even after introducing nonlocal perception term, the system remains stable; see Theorem 3.1 (i). What is worth mentioning is that if the system without nonlocal perception term is unstable, then under condition (C_2), the introduction of nonlocal perception term will make the system become stable when nonlocal perception coefficient α is greater than critical value α^*. See Theorem 3.1 (ii). This theoretical result is also very interesting to explain from a biological perspective. That is: when the random diffusion rate d_2 of the predator is greater than the random diffusion rate d_1 of the prey, a stronger nonlocal perception effect is beneficial for the stability of the system. This is also consistent with biological phenomena.

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