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Stability Analysis of a Diffusive Ratio-Dependent Predator-Prey Model with Nonlocal Perception*

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Abstract In this paper, we propose a diffusive ratio-dependent predator-prey model with a nonlocal perceptual term. We first discuss the well-posedness of solutions. Then for the model without the nonlocal perception term, we obtain stability conditions of the spatially homogeneous steady state. Under these conditions, when considering the effects of nonlocal perception, our result indicates that if the system without nonlocal perception term is stable, and even after introducing nonlocal perception term, the system remains stable. However, if the system without nonlocal perception term is unstable, and the introduction of nonlocal perception term will make the system become stable.

Keywords Predator-prey model, nonlocal perception, stability

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1. Introduction

Reaction-diffusion systems are crucial for understanding various phenomena in physics, chemistry, biology, and ecology. These systems typically describe the temporal and spatial evolution of concentrations of chemicals, biological populations, or other entities through reactions (interactions between species) and diffusion (propagation in space). It's known that spatial dispersion is a key factor contributing to the spatial heterogeneity that leads to the formation of spatial patterns. The significance of spatial models has been acknowledged by biologists for many years and has become a central topic in both ecology and mathematical ecology, see, e.g., [4, 17, 18].

A classic example is the Lotka-Volterra model, which has been extensively studied in the context of predator-prey dynamics. A traditional predator-prey model, which does not take into account spatial effects, can be described by the following system:

$$\begin{cases} \frac{dN}{dt} = rN(1 - \frac{N}{K}) - G(N, P)P, \\ \frac{dP}{dt} = \eta G(N, P)P - \gamma P, \end{cases} \tag{1.1}$$

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where N and P represent the population densities of prey and predators, respectively. The parameter $r > 0$ denotes the intrinsic growth rate of the prey, while K symbolizes the environmental carrying capacity. The function $G(N, P)$ means predation, referred to as the functional response. The parameter η represents the efficiency of converting biomass from predation, and γ stands for the per-capita death rate of the predators.

There are many works dedicated to studying (1.1), which indicates that ratio-dependent predation can support very rich dynamics. For example, when $G(N, P) = \frac{\alpha N}{N+aP}$, Chen et al. [9] explored complex dynamics in a ratio-dependent predator-prey model that includes the Allee effect and predator harvesting. When $G(N, P) = \frac{bN}{1+aN}$, Haque [12] showed complexity of the emergence of spatiotemporal within ratio-dependent predator-prey systems, focusing on patterns such as Turing patterns and wave bifurcations. And he concluded how spatial factors and predator-prey ratios can influence the overall dynamics of ecosystems. Lan et al. [14] investigated the dynamics of a ratio-dependent predator-prey model under the influence of stochastic environments, incorporating Holling III functional response and adding nonlinear harvesting terms. Tyutyunov and Titova [23] studied the dynamics of ratio-dependent predator-prey models with free boundaries, focusing on the long-term behaviors of both predator and prey populations. Xiao and Ruan [25] provided a comprehensive analysis of the global dynamics associated with ratio-dependent models. They addressed issues such as stability, bifurcations, and the conditions that lead to extinction or coexistence of species.

Arditi and Ginzburg [3] suggested the following ratio-dependent prey-predator model:

$$\begin{cases} \frac{dN}{dt} = rN(1 - \frac{N}{K}) - \frac{\alpha NP}{P+\alpha\beta N}, \\ \frac{dP}{dt} = \frac{\eta\alpha NP}{P+\alpha\beta N} - \gamma P, \end{cases} \tag{1.2}$$

where α and β represent the predator's attack rate and handing time, respectively. The biological meanings of other parameters in the model are the same as those in (1.1). Introducing the diffusion terms into system (1.2), the following ratio-dependent reaction-diffusion system is obtained:

$$\begin{cases} \frac{\partial N}{\partial t} = d_1 N_{xx} + rN(1 - \frac{N}{K}) - \frac{\alpha NP}{P+\alpha\beta N}, \\ \frac{\partial P}{\partial t} = d_2 P_{xx} + \frac{\eta\alpha NP}{P+\alpha\beta N} - \gamma P, \end{cases} \tag{1.3}$$

where d_1 and d_2 are the diffusion rates of the prey and predator.

By introducing the following transformations,

$$u = \frac{\alpha\beta}{\eta K}N, v = \frac{\alpha\beta}{\eta^2 K}P, \tilde{t} = \frac{\eta}{\beta}t, \hat{x} = \sqrt{\frac{\eta}{\beta}}x,$$

system (1.3) is simplified as

$$\begin{cases} \frac{\partial u}{\partial t} = d_1 u_{xx} + au(1 - \frac{u}{b}) - \frac{buv}{bu+v}, \\ \frac{\partial v}{\partial t} = d_2 v_{xx} - cv + \frac{buv}{bu+v}, \end{cases} \tag{1.4}$$

where the other nondimensional parameters are

$$a = \frac{r\beta}{\eta}, b = \frac{\alpha\beta}{\eta}, c = \frac{\gamma\beta}{\eta}.$$