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Nonlinear Dynamics and Pull-in Phenomena in a Magneto-Electro MEMS Actuator with Hardening Spring*

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Abstract This paper investigates the dynamic behavior and pull-in instability of a magneto-electro Micro-Electro-Mechanical System (MEMS) actuator, focusing on the nonlinear effects arising from magnetic forces and spring stiffness. The system consists of a movable wire attracted toward a stationary wire due to magnetic forces generated by applied currents. A critical equilibrium, known as the pull-in point, is reached when the currents exceed a threshold, leading to instability. We consider the governing equation based on Newton's Second Law, incorporating a nonlinear restoring force for the spring, which exhibits hardening behavior. The resulting second-order differential equation is analyzed using qualitative and bifurcation theories, revealing the critical bifurcation values determined by the currents and spring stiffness. Through a dynamical systems approach, we characterize the phase portraits and solutions, identifying distinct dynamical behaviors and the conditions for pull-in instability. Numerical simulations are performed to validate the analytical predictions, demonstrating excellent agreement with the theoretically derived threshold.

Keywords MEMS, magneto-electro actuator, nonlinear hardening spring, pull-in instability, bifurcation and qualitative analysis

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1. Introduction

Micro-Electro-Mechanical Systems (MEMS) are highly integrated devices that combine electrical, mechanical, and sensing components at the micro- or nano-scale, with typical dimensions and motion ranges measured in micrometers. As a key

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technology in modern micro- and nano-engineering, MEMS achieve miniaturization, high integration, and intelligent functionality. Unlike traditional mechanical engineering, MEMS fabrication relies on advanced semiconductor processes, such as photolithography, etching, and thin-film deposition, ensuring compatibility with integrated circuits and leveraging micro- and nano-fabrication techniques [1]. This approach enables MEMS to realize complex mechanical structures with high precision, reliability, and low power consumption.

Recent advancements in fabrication technologies have significantly expanded the capabilities of MEMS, enabling the development of novel microstructures such as micro-sensors, micro-actuators, and micro-resonators [2]. These innovations have enhanced MEMS performance in terms of accuracy, reliability, and adaptability, opening new application possibilities in biomedical engineering, aerospace, consumer electronics, and industrial automation [3, 4]. However, the dynamic behavior of MEMS devices often exhibits strong nonlinearities, such as nonlinear elastic forces, electrostatic coupling, and micro-scale damping effects [5, 6]. These nonlinear characteristics pose significant challenges to the dynamic response, stability, and control of MEMS. A thorough investigation into the dynamic properties of MEMS, including the mechanisms of nonlinear behavior and effective control strategies, is essential for optimizing performance and expanding application potential [7, 8].

The MEMS device under consideration consists of a stationary wire connected to a voltage source and a movable wire. When a voltage is applied, the movable wire is attracted toward the stationary one due to the magnetic force generated by the current. A critical equilibrium, known as the pull-in point, is reached when the currents in both wires exceed a threshold value. Beyond this threshold, if the current increases further, the movable wire will come into contact with the stationary wire, causing the microstructure to deviate from its optimal position. This phenomenon is referred to as pull-in instability, a significant nonlinear behavior in MEMS where the moving part approaches the actuating electrode. It is influenced by various parameters of the forces involved in the actuation and manipulation within the MEMS device [9, 10]. Nayfeh et al. [11] investigated pull-in instability in MEMS resonators, highlighting the distinct effects of Alternating Current (AC) and Direct Current (DC) loads. Their research, which focused on dynamic pull-in, provided essential guidelines for the safe design of MEMS resonant sensors. Zhang et al. [12] conducted a comprehensive review of the pull-in phenomenon in electrostatic MEMS and NEMS, elucidating the underlying physical principles responsible for instability and device failures. They also summarized the governing equations and conditions for predicting various pull-in behaviors. In recent years, the pull-in phenomenon in MEMS has garnered significant attention from researchers, who have employed diverse methodologies to explore its mechanisms and implications.

According to Newton's Second Law of Motion, the dynamics of a typical magneto-electro MEMS actuator can be described by the governing equation:

$$m\frac{d^2u}{dT^2} + F_s = F_e. \tag{1.1}$$

Here m is the mass of the movable current-carrying wire, u is the deformation length of the spring, F_s is the restoring force of the spring, and F_e represents the magnetic attraction force between the conductors.

The magnetic attraction force between the conductors due to the magnetic fields