Analysis of a Fractional Order Eco-Epidemiological Model with Prey Infection and Predator Cannibalism

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Abstract This study proposes a fractional-order prey-predator model that integrates disease dynamics within prey populations, fear induced behavioral responses, and predator cannibalism. The prey population is divided into susceptible and infected classes to represent disease dynamics effectively, while predator cannibalism is modeled as a survival strategy during prey scarcity. Essential properties such as non-negativity, boundedness, and the existence and uniqueness of solutions are thoroughly examined. Stability of equilibrium points is established, and global stability is proved using Lyapunov functions. Numerical simulations validate the theoretical results, highlighting how variations in cannibalism rates and fractional-order parameters influence population dynamics and system behavior.

Keywords Predator-prey system, Caputo fractional-order derivative, eco-epidemiological model, cannibalism, stability

MSC(2010) 92D40, 92D30, 92D25, 34D20, 34D23, 34D10

1. Introduction

The interaction between prey and predator is far more diversified and intricate than it might initially appear. Ecological dynamics encompass a wide range of phenomena such as parasitoidism [23], mutualism [21], prey-switching [8], trophic cascades [6], prey defences [45], prey's counter-attacks [20], and interspecies competition [44]. A particularly compelling aspect of predator-prey relationships is cannibalism, or intraspecific predation the act of killing and consuming individuals of the same species. Cannibalism is remarkably widespread in nature, documented in over 1300 species [36]. For example, male lions may consume unrelated cubs after

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taking control of a pride to maximize the reproductive success of their lineage [47]; octopuses are known to feed on their juveniles [9, 17]; and certain females consume non-viable or sick eggs to prevent infection [18]. This behavior is not limited to a narrow group: it occurs across a wide spectrum of species including fishes [16], frogs [32], insects [41], primates [15], and birds [19]. Drivers of cannibalism can include food scarcity, developmental stage, sex-based behaviors, famine, humidity, and other environmental or physiological pressures [36, 48]. These factors position cannibalism as an adaptive, though grim, strategy for survival and reproduction within the natural world.

In recent years, the profound influence of cannibalism on ecological systems has prompted researchers to incorporate it into mathematical models of population dynamics. Cannibalistic behavior has been modeled in predator species [31, 49], prey species [33, 37], and even within both populations simultaneously [4]. These models often extend to include additional ecological complexities such as time delays, infectious diseases, prey refuge mechanisms, and other modifying factors [27, 46, 50]. Analyses of such models reveal that cannibalism can exert either a stabilizing effect [46] or a destabilizing one [29] on population dynamics.

Cannibalism is sometimes referred to as the "lifeboat mechanism" because it can prevent the extinction of predator populations by providing an alternative energy source. Cannibalism and fear effects significantly influence predator-prey interactions. It allows predators to regulate their own populations, thereby reducing oscillations and promoting ecological stability. Fear effects, triggered by predators, alter prey behavior and limit their availability, which further impacts population dynamics and system balance. The combined impact of these factors creates a stabilizing force within ecosystems, as demonstrated in recent studies [7, 25, 31, 33, 50].

Species behavior is profoundly shaped by past experiences. For example, salamanders reduce foraging in the presence of garter snakes [28], while guppies modulate breeding based on predator-induced chemical cues [14]. Numerous adaptive responses such as altered foraging strategies, habitat preferences, and life-history traits emerge from these experiences and directly influence population dynamics [5]. In mathematical modeling, environmental parameters like reproduction and mortality rates vary over time, driven by psychological imprints retained by individuals. Recent experiences typically exert stronger influence due to the natural fading of older memories [10]. Capturing these evolving behavioral nuances is best achieved through fractional-order derivatives, which account for memory-dependent dynamics within interacting populations [40].

Fractional calculus has emerged as a powerful mathematical tool in recent years, finding widespread applications across diverse fields such as engineering, physics, economics, and applied sciences. Its significance lies in its ability to enhance the modeling and analysis of complex systems, especially in solving inverse problems that require fitting data with greater accuracy [3, 11, 22, 24, 38, 39, 42, 43]. Unlike classical calculus, fractional calculus introduces an additional parameter – the order of the derivative within mathematical models, providing a flexible and refined approach for data adjustment and prediction. By incorporating fractional derivatives, researchers can achieve improved alignment with real-world data, resulting in more reliable predictions for the evolution of dynamic systems. This approach has proven particularly valuable in applications such as disease spread modeling, where the incorporation of fractional-order enhances the predictive capabilities and accuracy of epidemiological models [11, 22, 38, 39, 42]. As fractional calculus contin-