

Mixed Convection Heat and Mass Transfer in a Micropolar Fluid with Soret and Dufour Effects

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Abstract. A mathematical model for the steady, mixed convection heat and mass transfer along a semi-infinite vertical plate embedded in a micropolar fluid in the presence of Soret and Dufour effects is presented. The non-linear governing equations and their associated boundary conditions are initially cast into dimensionless forms using local similarity transformations. The resulting system of equations is then solved numerically using the Keller-box method. The numerical results are compared and found to be in good agreement with previously published results as special cases of the present investigation. The non-dimensional velocity, microrotation, temperature and concentration profiles are displayed graphically for different values of coupling number, Soret and Dufour numbers. In addition, the skin-friction coefficient, the Nusselt number and Sherwood number are shown in a tabular form.

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Key words: Mixed convection, heat and mass transfer, micropolar fluid, Soret and Dufour effects.

1 Introduction

Flows in which the acceleration forces are large in comparison with the viscous forces, or the diffusion times are large in comparison with the convection times, or the convection velocities are large in comparison with the diffusion velocities, it can be shown that, e.g., the influences of wall boundaries on flows are restricted to small region (thin layer) near the walls. This thin layer where friction effects cannot be ignored is called the boundary layer. Such flows can therefore be subdivided into body-near regions, where viscous influences on flows have to be considered, and regions that are distant from the wall, which can be regarded as being free from viscous influences. This

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boundary layer theory was proposed by Ludwig Prandtl in 1904. Prandtl concluded that it might be sufficient in an analysis of a flow field to consider action of viscosity within these boundary layers, whereas the flow outside the boundary layers may be considered inviscid. He then proceeded to simplify the conservation equation by estimating the order of magnitude of the various terms in the conservation equations, and thus he derived the so-called boundary layer equations (Eckert and Drake, see [1]). Analytical treatment of the Navier-Stokes equations present great difficulties even in the case of steady two-dimensional incompressible flows. Only a limited number of exact solutions are known to exist for some special cases of these equations. An important contribution by Prandtl was to show that the Navier-Stokes equations can be simplified to yield an approximate set of boundary layer equations (Bejan, see [2]).

The analysis of mixed convection boundary layer flow along a vertical plate embedded in viscous fluid has received considerable theoretical and practical interest. The phenomenon of mixed convection occurs in many technical and industrial problems such as electronic devices cooled by fans, nuclear reactors cooled during an emergency shutdown, a heat exchanger placed in a low-velocity environment, solar collectors and so on. Several authors have studied the problem of mixed convection about different surface geometries. When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has been observed that an energy flux can be generated not only by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion-thermo (Dufour) effect. On the other hand, mass fluxes can also be created by temperature gradients and this embodies the thermal-diffusion (Soret) effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fouriers and Ficks laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. The Soret effect, for instance, has been utilized for isotope separation and in mixture between gases with very light molecular weight and of medium molecular weight. The Dufour effect was recently found to be of order of considerable magnitude so that it cannot be neglected (see Eckert and Drake [1]). Dursunkaya and Worek [3] studied diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface, whereas Kafoussias and Williams [4] presented the same effects on mixed convective and mass transfer transfer steady laminar boundary layer flow over a vertical flat plate with temperature dependent viscosity. Postelnicu [5] studied numerically the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects. Both free and forced convection boundary layer flows with Soret and Dufour effects have been addressed by Abreu et al. [6]. Alam and Rahman [7] have investigated the Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction. Recently, the effect of Soret and Dufour parameters on free convection heat and mass transfers from a vertical surface in a doubly stratified Darcian

porous medium has been reported by Lakshmi Narayana and Murthy [8].

The study of non-Newtonian fluid flows has gained much attention from the researchers because of its applications in biology, physiology, technology and industry. In addition, the effects of heat and mass transfer in non-Newtonian fluid also have great importance in engineering applications like the thermal design of industrial equipment dealing with molten plastics, polymeric liquids, foodstuffs, or slurries. Several investigators have extended many of the available convection heat and mass transfer problems to include the non-Newtonian effects. Many of the non-Newtonian fluid models describe the nonlinear relationship between stress and the rate of strain. But the micropolar fluid model introduced by Eringen [9] exhibits some microscopic effects arising from the local structure and micro motion of the fluid elements. Further, the micropolar fluid can sustain couple stresses and include classical Newtonian fluid as a special case. The model of micropolar fluid represents fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. Micropolar fluids have been shown to accurately simulate the flow characteristics of polymeric additives, geomorphological sediments, colloidal suspensions, haematological suspensions, liquid crystals, lubricants etc. The mathematical theory of equations of micropolar fluids and applications of these fluids in the theory of lubrication and porous media are presented by Lukaszewicz [10]. The heat and mass transfer in micropolar fluids is also important in the context of chemical engineering, aerospace engineering and also industrial manufacturing processes. The problem of mixed convection heat and mass transfer in the boundary layer flow along a vertical surface submerged in a micropolar fluid has been studied by a number of investigators. Ahmadi [11] has studied the boundary layer flow of a micropolar fluid over a semi-infinite plate. Laminar mixed convection boundary layer flow of a micropolar fluid from an isothermal vertical flat plate has been considered by Jena and Mathur [12]. Asymptotic boundary layer solutions are presented for the combined convection from a vertical semi-infinite plate to a micropolar fluid by Gorla et al. [13]. Tian-Yih Wang [14] have examined the effect of wall conduction on laminar mixed convection heat transfer of micropolar fluids along a vertical flat plate. Although the Soret and Dufour effects of the medium on the heat and mass transfer in a micropolar fluid are important, very little work has been reported in the literature. Beg et al. [15] have analyzed the two dimensional coupled heat and mass transfer of an incompressible micropolar fluid past a moving vertical surface embedded in a Darcy-Forchheimer porous medium in the presence of Soret and Dufour effects. A mathematical model for the steady thermal convection heat and mass transfer in a micropolar fluid saturated Darcian porous medium in the presence of Dufour and Soret effects and viscous heating is presented by Rawat and Bhargava [16].

Motivated by the investigations mentioned above, the purpose of the present work is to investigate the Soret and Dufour effects on mixed convection heat and mass transfer along a semi-infinite vertical plate in a micropolar fluid with uniform wall temperature and concentration. The Keller-box method given in [17] and [18] is employed to

solve the non-linear system in the problem. The effects of micropolar parameter, Soret and Dufour numbers are examined and are displayed through graphs. The results are compared with relevant results in the existing literature and are found to be in good agreement.

2 Mathematical formulation

Consider a steady, two-dimensional mixed convective heat and mass transfer along a semi-infinite vertical plate embedded in a free stream of micropolar fluid with velocity u_∞ , temperature T_∞ and concentration C_∞ . Choose the coordinate system such that x -axis is along the vertical plate and y -axis normal to the plate. The physical model and coordinate system are shown in Fig. 1. The plate is maintained at uniform wall temperature and concentration T_w and C_w respectively. These values are assumed to be greater than the ambient temperature T_∞ and concentration C_∞ at any arbitrary reference point in the medium (inside the boundary layer). In addition, the Soret and Dufour effects are considered.

Using the Boussinesq and boundary layer approximations, the governing equations for the micropolar fluid in the presence of Soret and Dufour effects (see [11–13] and [19]) are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (2.1a)$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = (\mu + \kappa) \frac{\partial^2 u}{\partial y^2} + \kappa \frac{\partial \omega}{\partial y} + \rho g^* (\beta_T (T - T_\infty) + \beta_C (C - C_\infty)), \quad (2.1b)$$

$$\rho j \left(u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} \right) = \gamma \frac{\partial^2 \omega}{\partial y^2} - \kappa \left(2\omega + \frac{\partial u}{\partial y} \right), \quad (2.1c)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{DK_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2}, \quad (2.1d)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + \frac{DK_T}{T_m} \frac{\partial^2 T}{\partial y^2}, \quad (2.1e)$$

where u and v are the velocity components in x and y directions respectively, ω is the component of microrotation whose direction of rotation lies in the xy -plane, T is the temperature, C is the concentration, g^* is the acceleration due to gravity, ρ is the density, μ is the dynamic coefficient of viscosity, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansions, κ is the vortex viscosity, j is the micro-inertia density, γ is the spin-gradient viscosity, α is the thermal diffusivity, D is the solutal diffusivity of the medium, C_p is the specific heat capacity, C_s is the concentration susceptibility, T_m is the mean fluid temperature and K_T is the thermal diffusion ratio. The last term on the right-hand side of the energy equation (2.1d) and diffusion equation (2.1e) signifies the Dufour or diffusion-thermo effect and the Soret or thermal-diffusion effect, respectively [19].

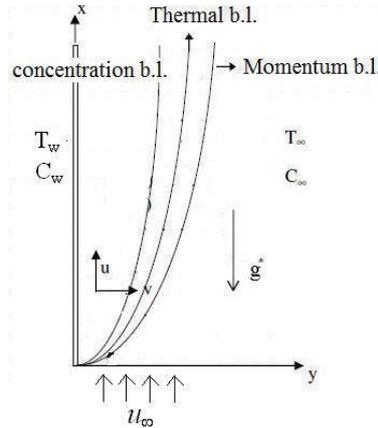


Figure 1: Physical model and coordinate system.

The boundary conditions are

$$u = 0, \quad v = 0, \quad \omega = 0, \quad T = T_w, \quad C = C_w, \quad \text{at } y = 0, \quad (2.2a)$$

$$u = u_\infty, \quad \omega = 0, \quad T = T_\infty, \quad C = C_\infty, \quad \text{as } y \rightarrow \infty, \quad (2.2b)$$

where the subscripts w and ∞ indicate the conditions at the wall and at the outer edge of the boundary layer respectively and k is the thermal conductivity of the fluid. The boundary condition $\omega=0$ in Eq. (2.2a), represents the case of concentrated particle flows in which the microelements close to the wall are not able to rotate.

In view of the continuity equation (2.1a), we introduce the stream function ψ by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \quad (2.3)$$

Substituting (2.3) in (2.1b)-(2.1e) and then using the following local similarity transformations

$$\eta = \frac{y}{x} Re_x^{\frac{1}{2}}, \quad \psi = \nu Re_x^{\frac{1}{2}} f(\eta), \quad \omega = \frac{\nu}{x^2} Re_x^{\frac{3}{2}} g(\eta), \quad (2.4a)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad (2.4b)$$

we get the following local similarity equations

$$\left(\frac{1}{1-N}\right) f''' + \frac{1}{2} f f'' + \left(\frac{N}{1-N}\right) g' + g_s \theta + g_c \phi = 0, \quad (2.5a)$$

$$\lambda g'' + \frac{1}{2} f g' + \frac{1}{2} f' g - \left(\frac{N}{1-N}\right) \mathcal{J}(2g + f'') = 0, \quad (2.5b)$$

$$\frac{1}{Pr} \theta'' + \frac{1}{2} f \theta' + D_f \phi'' = 0, \quad (2.5c)$$

$$\frac{1}{Sc} \phi'' + \frac{1}{2} f \phi' + S_r \theta'' = 0, \quad (2.5d)$$

where the primes indicate partial differentiation with respect to η alone, ν is the kinematic viscosity, $Gr_x = g^* \beta_T (T_w - T_\infty) x^3 / \nu^2$ is the local thermal Grashof number, $Gc_x = g^* \beta_C (C_w - C_\infty) x^3 / \nu^2$ is the local solutal Grashof number, $Re_x = u_\infty x / \nu$ is the local Reynolds number, $g_s = Gr_x / Re_x^2$ is the temperature buoyancy parameter, $g_c = Gc_x / Re_x^2$ is the mass buoyancy parameter, $Pr = \nu / \alpha$ is the Prandtl number, $Sc = \nu / D$ is the Schmidt number, $\mathcal{J} = x\nu / [ju_\infty]$ is the micro-inertia density and $\lambda = \gamma / [j\rho\nu]$ is the spin-gradient viscosity, $N = \kappa / (\mu + \kappa)$ ($0 \leq N < 1$) is the Coupling number [20], $D_f = DK_T (C_w - C_\infty) / [C_s C_p \nu (T_w - T_\infty)]$ is the Dufour number and $S_r = DK_T (T_w - T_\infty) / [T_m \nu (C_w - C_\infty)]$ is the Soret number.

Boundary conditions (2.2) in terms of f, g, θ and ϕ become

$$\eta = 0 : \quad f(0) = 0, \quad f'(0) = 0, \quad g(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1, \quad (2.6a)$$

$$\eta \rightarrow \infty : \quad f'(\infty) = 1, \quad g(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0. \quad (2.6b)$$

If $D_f = 0$ and $S_r = 0$, the problem reduces to mixed convection heat and mass transfer in a micropolar fluid without Soret and Dufour effects. Also, in the limit as $N \rightarrow 0$, the governing equations (2.1a)-(2.1e) reduce to the corresponding equations for a mixed convection heat and mass transfer in a viscous fluid. Hence, the case of combined free-forced convective heat and mass transfer on a semi-infinite vertical plate of Kafousias [21] can be obtained by taking $N = 0$, $D_f = 0$ and $S_r = 0$.

The wall shear stress, heat and mass transfers from the plate respectively are given by

$$\tau_w = \left[(\mu + \kappa) \frac{\partial u}{\partial y} + \kappa \omega \right]_{y=0}, \quad q_w = -k \left[\frac{\partial T}{\partial y} \right]_{y=0}, \quad q_m = -D \left[\frac{\partial C}{\partial y} \right]_{y=0}. \quad (2.7)$$

The non-dimensional skin friction $C_f = 2\tau_w / [\rho U_*^2]$, the local Nusselt number $Nu_x = q_w x / [k(T_w - T_\infty)]$ and local Sherwood number $Sh_x = q_m x / [D(C_w - C_\infty)]$, where U_* is the characteristic velocity, are given by

$$C_f Re_x^{\frac{1}{2}} = \left(\frac{2}{1-N} \right) f''(0), \quad \frac{Nu_x}{Re_x^{\frac{1}{2}}} = -\theta'(0), \quad \frac{Sh_x}{Re_x^{\frac{1}{2}}} = -\phi'(0). \quad (2.8)$$

3 Results and discussions

The system of non-linear ordinary differential equations (2.5a)-(2.5d) together with the boundary conditions (2.6) are locally similar and solved numerically using Keller-box implicit method discussed in [17, 18], and in the review paper by Keller [22]. This method has been successfully used by several authors to study boundary layer flows (see [24, 25] and [23]). The method has the following four main steps:

Step 1 Reduce the system of Eqs. (2.5a) to (2.5d) to a first order system;

Step 2 Write the difference equations using central differences;

Step 3 Linearize the resulting algebraic equations by Newtons method and write them in matrix-vector form;

Step 4 Use the block-tridiagonal-elimination technique to solve the linear system.

This method has been proven to be adequate and give accurate results for boundary layer equations. A uniform grid was adopted, which is concentrated towards the wall. The calculations are repeated until some convergent criterion is satisfied and the calculations are stopped when

$$\delta f_0'' \leq 10^{-8}, \quad \delta g_0' \leq 10^{-8}, \quad \delta \theta_0' \leq 10^{-8}, \quad \text{and} \quad \delta \phi_0' \leq 10^{-8}.$$

In the present study, the boundary conditions for η at ∞ are replaced by a sufficiently large value of η where the velocity approaches one and microrotation, temperature and concentration approach zero. In order to see the effects of step size ($\Delta\eta$) we ran the code for our model with three different step sizes as $\Delta\eta=0.001$, $\Delta\eta=0.01$ and $\Delta\eta=0.05$ and in each case we found very good agreement between them on different profiles. After some trials we imposed a maximal value of η at ∞ of 10 and a grid size of η as 0.01.

In order to study the effects of micropolar parameter N , Soret number S_r and Dufour number D_f explicitly, computations were carried out for the cases of $g_s=1.0$ and $g_c=0.1$, $Pr=0.71$, $Sc=0.22$. The values of Soret number S_r and Dufour number D_f are chosen in such a way that their product is constant according to their definition, provided that the mean temperature T_m is constant. The values of micropolar parameters $\mathcal{J}=0.1$ and $\lambda=1.0$ are chosen so as to satisfy the thermodynamic restrictions on the material parameters given by Eringen [9].

In the absence of coupling number N , Soret number S_r and Dufour number D_f with $\mathcal{J}=0$, $\lambda=0$, $Pr=0.73$ and $Sc=0.24$ for different values of buoyancy parameters g_s and g_c , the results have been compared with the special case Kafoussias [21] and found that they are in good agreement, as shown in Table 1.

In Figs. 2-5, the effects of the coupling number N on the dimensionless velocity, microrotation, temperature and concentration are presented for fixed values of Soret and

Table 1: Comparison of results for a vertical plate in viscous fluids without Soret and Dufour effects case [21].

| | | $f''(0)$ | | $-\theta'(0)$ | |
|-------|-------|------------|---------|---------------|---------|
| g_s | g_c | Kafoussias | Present | Kafoussias | Present |
| 0.1 | 0.05 | 0.5538 | 0.5538 | 0.3296 | 0.3296 |
| 0.1 | 0.10 | 0.6317 | 0.6317 | 0.3404 | 0.3404 |
| 0.1 | 0.20 | 0.7776 | 0.7776 | 0.3589 | 0.3589 |
| 1.0 | 0.05 | 1.4452 | 1.4452 | 0.4129 | 0.4129 |
| 1.0 | 0.10 | 1.5007 | 1.5007 | 0.4179 | 0.4179 |
| 1.0 | 0.20 | 1.6096 | 1.6096 | 0.4274 | 0.4274 |
| 10.0 | 0.05 | 6.8389 | 6.8389 | 0.6449 | 0.6449 |
| 10.0 | 0.10 | 6.8715 | 6.8714 | 0.6461 | 0.6462 |
| 10.0 | 0.20 | 6.9366 | 6.9363 | 0.6487 | 0.6488 |

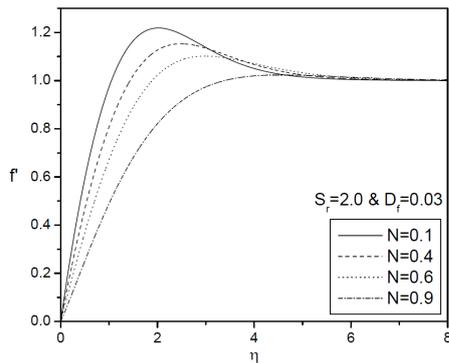


Figure 2: Velocity profiles for various values of N .

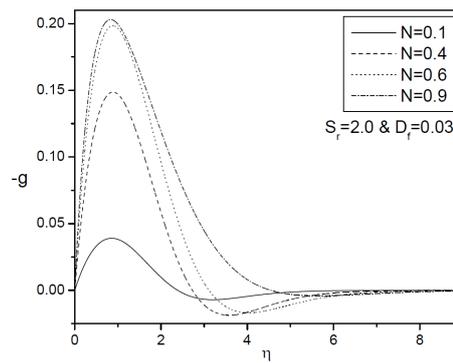


Figure 3: Microrotation profiles for various values of N .

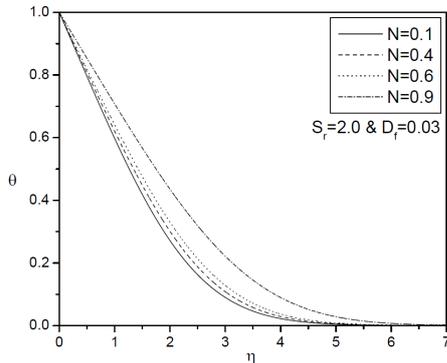


Figure 4: Temperature profiles for various values of N .

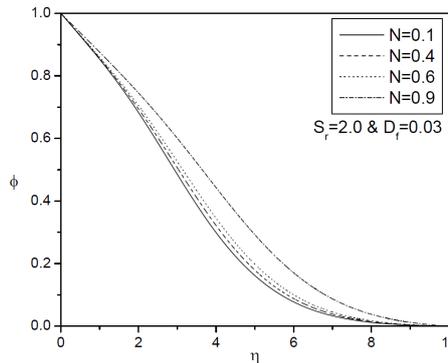


Figure 5: Concentration profiles for various values of N .

Dufour numbers. As N increases, it can be observed from Fig. 2 that the maximum velocity decreases in amplitude and the location of the maximum velocity moves farther away from the wall. Since $N \rightarrow 0$ corresponds to viscous fluid, the velocity in case of micropolar fluid is less compared to that of viscous fluid case. From Fig. 3, it is shown that, as N increase the angular velocity profiles decrease beside the vertical plate and increase far away from the vertical plate. As $N \rightarrow 0$, the microrotation tends to zero because in the limit N tends to zero the micro-polarity is lost and the fluid behaves as non-polar fluid. It is clear from Fig. 4 that the temperature increases with the increase of coupling number N . It can be seen from Fig. 5 that the concentration of the fluid increases with the increase of coupling number N . The temperature and concentration in case of micropolar fluids is more than that of the Newtonian fluid case.

Fig. 6 displays the non-dimensional velocity for different values of Soret number S_r and Dufour number D_f with fixed value of coupling number N . It is observed that the velocity of the fluid increases with the increase of Dufour number (or decrease of Soret number). It can be observed from Fig. 7 that the microrotation component decrease near the vertical plate and increase far away from the plate with increasing Dufour number (or decreasing of Soret number), showing a reverse rotation near the two boundaries. The reason is that the microrotation field in this region is dominated by a small number of particles spins that are generated by collisions with the

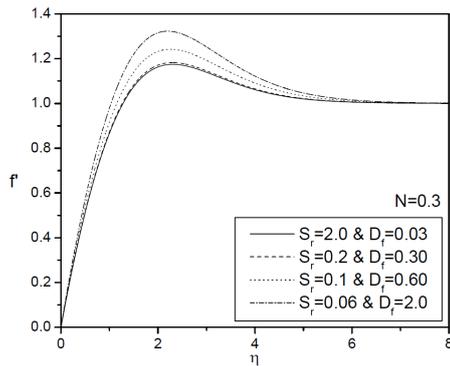


Figure 6: Velocity profiles for various values of S_r and D_f .

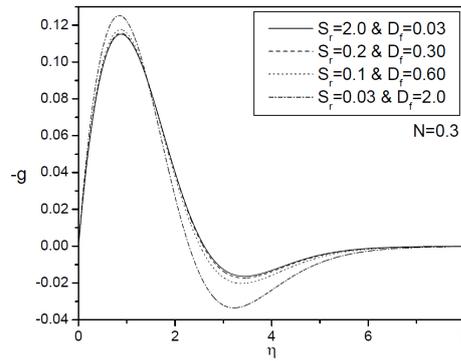


Figure 7: Microrotation profiles for various values of S_r and D_f .

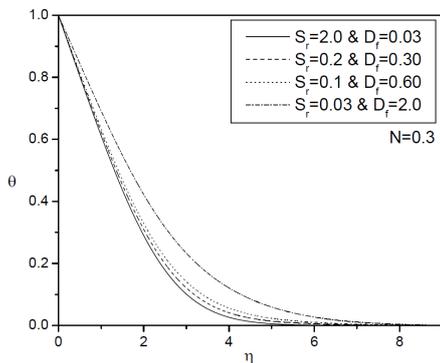


Figure 8: Temperature profiles for various values of S_r and D_f .

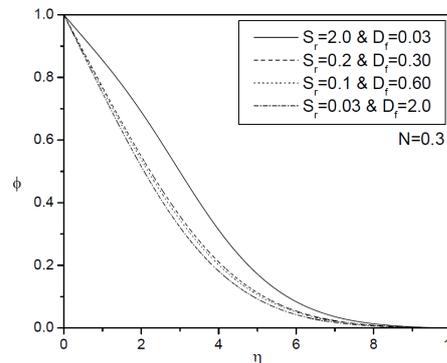


Figure 9: Concentration profiles for various values of S_r and D_f .

boundary. The dimensionless temperature for different values of Soret number S_r and Dufour number D_f for $N=0.3$, is shown in Fig. 8. It is clear that the temperature of the fluid increases with the increase of Dufour number (or decrease Soret number). Fig. 9 demonstrates the dimensionless concentration for different values of Soret number S_r and Dufour number D_f for $N=0.3$. It is seen that the concentration of the fluid decreases with increase of Dufour number (or decrease of Soret number).

The variations of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ which are proportional to the local skin-friction coefficient, rate of heat and mass transfers are shown in Table 2 for different values of the coupling number with fixed $S_r=2.0$ and $D_f=0.03$. From this table it is observed that the the value of $f''(0)$ decreases with the increasing values of coupling number. Also, these values (local viscous drag) are higher for the Newtonian fluid ($N=0$) than the micropolar fluid ($N \neq 0$). The heat and mass transfer rates decrease with the increasing values of coupling number. From this data, it is obvious that micropolar fluids present lower heat and mass transfer values than those of Newtonian fluids. Since the skin-friction coefficient as well as heat and mass transfers are lower in the micropolar fluid comparing to the Newtonian fluid, which may be beneficial in flow, temperature and concentration control of polymer processing. Finally, the effects of Dufour and Soret number on the local skin-friction coefficient and the rate of heat

Table 2: Effects of skin friction, heat and mass transfer coefficients for varying values of coupling, Soret and Dufour numbers.

| N | S_r | D_f | $f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|-----|-------|--------|----------|---------------|-------------|
| 0.1 | 2.0 | 0.03 | 1.42349 | 0.40971 | 0.14463 |
| 0.2 | 2.0 | 0.03 | 1.31175 | 0.40212 | 0.14390 |
| 0.3 | 2.0 | 0.03 | 1.19206 | 0.39385 | 0.14305 |
| 0.4 | 2.0 | 0.03 | 1.06430 | 0.38470 | 0.14204 |
| 0.5 | 2.0 | 0.03 | 0.92832 | 0.37437 | 0.14078 |
| 0.6 | 2.0 | 0.03 | 0.78382 | 0.36230 | 0.13909 |
| 0.7 | 2.0 | 0.03 | 0.62996 | 0.34740 | 0.13660 |
| 0.8 | 2.0 | 0.03 | 0.46447 | 0.32716 | 0.13242 |
| 0.9 | 2.0 | 0.03 | 0.28013 | 0.29360 | 0.12368 |
| 0.2 | 2.0 | 0.03 | 1.31175 | 0.40212 | 0.14390 |
| 0.2 | 1.6 | 0.0375 | 1.30927 | 0.40135 | 0.16397 |
| 0.2 | 1.2 | 0.05 | 1.30717 | 0.40040 | 0.18402 |
| 0.2 | 1.0 | 0.06 | 1.30641 | 0.39977 | 0.19406 |
| 0.2 | 0.8 | 0.075 | 1.30602 | 0.39896 | 0.20412 |
| 0.2 | 0.5 | 0.12 | 1.30717 | 0.39686 | 0.21935 |
| 0.2 | 0.2 | 0.3 | 1.31861 | 0.38950 | 0.23553 |
| 0.2 | 0.1 | 0.6 | 1.34062 | 0.37741 | 0.24260 |

and mass transfer are also shown in this table. The behavior of these parameters is self-evident from the Table 2 and hence are not discussed for brevity.

4 Conclusions

In this paper, a boundary layer analysis for mixed convection heat and mass transfer in a micropolar fluid over a vertical plate with uniform wall temperature and concentration conditions in the presence of Soret and Dufour effects is considered. Using the similarity variables, the governing equations are transformed into a set of non-similar parabolic equations and numerical solution for these equations has been presented for different values of parameters. The higher values of the coupling number N (i.e., the effect of microrotation becomes significant) result in lower velocity distribution but higher wall temperature; wall concentration distributions in the boundary layer compared to the Newtonian fluid case. The numerical results indicate that the skin friction coefficient as well as rate of heat and mass transfers in the micropolar fluid are lower compared to that of the Newtonian fluid. The present analysis has also shown that the flow field is appreciably influenced by the Dufour and Soret effects.

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