Journal of Nonlinear Modeling and Analysis http://jnma.ca; http://jnma.ijournal.cn
Volume 4, Number 3, September 2022, 562–586 DOI:10.12150/jnma.2022.562

Stability of Delayed Markovian Switching Stochastic Neutral-type Reaction-diffusion Neural Networks*

Suriguga[1], Chuntao Shao[2,†] and Yonggui Kao[1]

Abstract This paper is concerned about exponential stability in mean square of Markovian switching delayed reaction-diffusion neutral-type stochastic neural networks (RNSNNs). By Lyapunov function method, several novel stability criteria on exponential mean square stability of Markovian switching RNSNNs with time-varying delays are obtained. In the end, two examples are given to verify the feasibility of our findings.

Keywords Markovian switching, Neutral-type, Neural network, Reaction-diffusion system.

MSC(2010) 34D99, 34F05, 35K57.

1. Introduction

Many practical systems are often disturbed by various kinds of noise. When a system is affected by some random factors in the environment, the phenomenon can no longer be described by definite differential equations, so the stochastic differential equation appears. For example, the change of stock price in financial market and conservative mechanical system can be described by stochastic differential equations. Based on G-Lyapunov functional method, the criteria of quasi-surely exponential stability and finite-time stability of stochastic reaction-diffusion systems driven by G-Brownian motion have been established [18]. [17] aimed to design feedback control based on past state to stabilize a class of nonlinear stochastic differential equations driven by G-Brownian motion. The stability of stochastic differential equations driven by G-Brownian motion via feedback control based on discrete time state observation has been studied [25].

In the last decades, a growing number of scholars have been inclined to the study of stochastic partial differential equations (SPDEs). SPDEs have been used to describe a large number of mathematical models in many subjects such as biology and physics. Kao et al., explored the stability of coupled SRDSs on networks

†the corresponding author.
Email address: shaw@hitwh.edu.cn (C. Shao), kaoyonggui@sina.com (Y. Kao), m15248082959@163.com (Suriguga)
[1]Department of Mathematics, Harbin Institute of Technology (Weihai), Weihai, Shandong 264209, China
[2]Department of Control Science and Engineering, Harbin Institute of Technology (Weihai), Shandong 264209, China
*The authors were supported by Natural Science Foundations of China (No. 61873071) and Shandong Major Projects of Natural Sciences Foundation (SPNSF) (Grant Nos. ZR2020ZD27, ZR2019MF006).

with Markovian switching by constructing the Lyapunov function [10]. Kao et al., employed Itô formula studying the stability in mean of SRDSs with Markovian switching [11]. By using Leary Schauder fixed point theorem and Lyapunov method, the existence of solutions and the global Mittag-Leffler stability criterion for fractional coupled reaction-diffusion NNs with delay without strong connectivity have been given [2]. Applying Lyapunov-Krasovskii (LK) functional, stochastic analysis technology and LMIs technique, reaction-diffusion statistic CGNNs (RDSCGNNs) with mixed time delays have been discussed [9, 27] respectively. The global existence, uniqueness, uniform boundedness and asymptotic behavior of solutions for a weakly coupled reaction-diffusion systems have been studied [5].

In addition, there are many structurally mutated systems in the real world, such as computer control system, chemical process and communication system, which can be described by Markovian jumping system. Markovian switching system is a hybrid system consisting of discrete time state and continuous time state. The delay-dependent stabilization problem for a class of stochastic Markov systems with event-triggered feedback control was studied [39]. Exponential stability and instability of Markovian switching impulsive stochastic functional differential equations (SFDSs) were discussed by Lyapunov direct method [13]. Mean square exponential stability of Markovian jumping time-varying delayed reaction-diffusion HNNs with uncertain transition rates by Lyapunov-Krasovskii functional method and linear matrix inequality [30].

In the real world, stochastic differential equations with time delay have important applications in biological engineering, ship stability control and other fields. If a system depends not only on the present state and the past state, but also on the rate of state change in the past period of time, delayed neutral stochastic differential equation is used to describe the system. The delayed neutral stochastic differential equation has certain application value in the fields of economics, biology, mechanics and electronics. Mean square exponential stability criteria of mixed delayed impulsive fuzzy RDSCGNNs were derived mainly by "M-cone" approach and LMI techniques [32]. A low conservative criterion for asymptotic stability of a class of fractional neutral-type delayed NNs with Riemann-Liouville meaning have been considered [38]. The problem of distributed event-triggered control for nonlinear stochastic multi-agent systems with external disturbance and time delay have been discussed [29]. The exponential synchronization problem of time-varying delayed coupled stochastic reaction-diffusion NNs were studied [37]. Global exponential stability and instability criteria of impulsive SFDSs have been obtained [4]. Besides, stability of impulsive stochastic systems have been studied [35, 36] respectively. Furthermore, many efforts have been done in this area. Some results on the global stochastic exponential stability, almost sure exponential stability, mean value exponential stability and mean square exponential stability have been obtained, and please refer to [1, 22, 31, 34]. However, very few results have been reported on mean square exponential stability of time-varying delayed Markovian switching reaction-diffusion neutral-type stochastic neural networks.

Based on the above concerns, we discuss stability criteria for time-varying delayed RNSNNs with Markovian switching as follows. Section 2 introduces several lemmas and definitions. Section 3 presents exponential mean square stability criteria of Markovian switching time-varying delayed RNSNNs. Section 4 illustrates the effectiveness of our findings. Finally, a conclusion is presented.

Notation: We assume that $G = \{x||x_l| < d_l, l = 1, \cdots, n\}$ is a bounded set of $\mathbb{R}^n$

with smooth boundary ∂G, $\mu(G)>0$; $C=C([-\varrho,0]\times G;\mathbb{R}^m)$ indicates the set of all continuous $\mathbb{R}^m$-valued function ψ on $[-\varrho,0]\times G$ with $\|\psi\|_C=\sup\limits_{\vartheta\in[-\varrho,0]}|\psi(\vartheta,x)|$, where $\varrho=\max\{\varrho_1,\varrho_2\}$. Denoted by $L^2_{\mathcal{F}_t}=L^2_{\mathcal{F}_t}([-\varrho,0]\times G;\mathbb{R}^m)$, the set of all $\mathcal{F}_t$-measurable, C-valued stochastic variable ψ with $E\|\psi\|^2<\infty$.

For $y(t,x)=(y_1(t,x),\cdots,y_m(t,x))^{\mathrm{T}}\in\mathbb{R}^m$, we define

$$\|y(t,x)\|_2=\sum_{k=1}^m\|y_k(t,x)\|_2=\sum_{k=1}^m\Big(\int_G|y_k(t,x)|^2dx\Big)^{\frac{1}{2}},\tag{1.1}$$

and for any $\psi(\vartheta,x)=(\psi_1(\vartheta,x),\cdots,\psi_m(\vartheta,x))^{\mathrm{T}}\in C$, the norm on C is defined by

$$\|\psi(t,x)\|_2=\sup_{-\varrho\le\vartheta\le0}\sum_{k=1}^m\|\psi_k(t,x)\|_2.\tag{1.2}$$

Then, it can be proved that C is a Banach space. Besides, in the next few sections, we will employ the following abbreviations

$$\xi=\xi(t,x),\ \xi_{\varrho_1}=\xi(t-\varrho_1(t),x),\ \xi_{\varrho_2}=\xi(t-\varrho_2(t),x)\ (\xi=y,u).$$

2. Preliminaries

Suppose that $|\cdot|$ be the Euclidean norm in $\mathbb{R}^m$, $\mathbb{R}^+=[0,\infty)$; $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\ge0},P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\ge0}$ satisfying usual conditions; $r(t)$ $(t\ge0)$ be a Markov chain on $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\ge0},P)$, which takes values in a finite state space $S=\{1,\ldots\ldots,N\}$ with generator $\Pi=\{\pi_{ij}\}(i,j\in S)$ shown as

$$P(r(t+\Delta)=j|r(t)=i)=\begin{cases}\pi_{ij}\Delta+o(\Delta), & if\quad i\neq j,\\ 1+\pi_{ii}\Delta+o(\Delta), & if\quad i=j,\end{cases}$$

where $\Delta>0$ and $\lim\limits_{\Delta\to0}\frac{o(\Delta)}{\Delta}=0$, $\pi_{ij}\ge0$ is the transition rate from i to j if $i\neq j$, and $\pi_{ii}=-\sum_{j\neq i}\pi_{ij}$.

A kind of RNSNNs with time-varying delayed Markovian switching parameters are considered, which has the following form

$$\begin{cases}\dfrac{\partial y}{\partial t}=\displaystyle\sum_{l=1}^n\frac{\partial}{\partial x_l}\Big(\alpha_l(t,x,y)\frac{\partial y}{\partial x_l}\Big)-A(r(t))y+B(r(t))f(y)+C(r(t))f(y_{\varrho_1})\\ \qquad\qquad+D(r(t))\dfrac{\partial y_{\varrho_2}}{\partial t}+J+g(t,x,y,y_{\varrho_1},y_{\varrho_2},r(t))\dot{w}(t),\quad t\ge t_0,\ x\in G,\ r(t)\in S,\\ y(\vartheta,x)=\psi(\vartheta,x),\quad \vartheta\in[-\varrho,0],\quad x\in G,\\ \dfrac{\partial y}{\partial\mathcal{N}}=0,\quad t\in[0,+\infty),\ x\in\partial G,\end{cases}\tag{2.1}$$

where $y=[y_1,\cdots,y_m]^{\mathrm{T}}\in\mathbb{R}^m$ is the state vector associated with m neurons at time t; $x=(x_1,x_2,\cdots,x_n)^{\mathrm{T}}\in G\subset\mathbb{R}^n$, $\mathcal{N}$ is the unit normal vector of ∂G; $w(t)$ is a m-Brownian motion; $r(t)$ and $w(t)$ are independent; $\alpha_l(t,x,y)=\mathrm{diag}(\alpha_{1l},\alpha_{2l},\cdots,\alpha_{ml})$ is the transmission diffusion coefficient along the neurons, and $\alpha_{kl} = \alpha_{kl}(t,x,y) \geq 0$ $(k = 1,\cdots,m)$; $D(r(t)) = (d_{kj}(r(t)))_{m\times m}$, $A(r(t)) = \mathrm{diag}\{a_1(r(t),\cdots,a_m(r(t))\}$ with positive entries $a_k(r(t)) > 0$ $(k = 1,\cdots,m)$, $B(r(t)) = (b_{kj}(r(t)))_{m\times m}$ is a connection weight matrix, and $C(r(t)) = (c_{kj}(r(t)))_{m\times m}$ is a discrete time-varying delay connection weight matrix; $J = [J_1,\cdots,J_m]^{\mathrm{T}} \in \mathbb{R}^m$ is a constant external input vector; $f(y) = [f_1(y_1),\cdots,f_m(y_m)]^{\mathrm{T}} \in \mathbb{R}^m$ is the nonlinear neuron activation function which describes the behavior in which the neurons respond to each other; $\varrho_1(t)$ is a discrete time-varying delay, $\varrho_2(t)$ is a neutral time-varying delay, and satisfy $0 \leq \varrho_0 \leq \varrho_1(t) \leq \varrho_1$, $\dot{\varrho}_1(t) \leq \kappa_1$ and $0 \leq \varrho_2(t) \leq \varrho_2$, $\dot{\varrho}_2(t) \leq \kappa_2$; The noise perturbation $g : \mathbb{R}^+ \times G \times \mathbb{R}^m \times \mathbb{R}^m \times \mathbb{R}^m \times S \to \mathbb{R}^{m\times m}$ is a Borel measurable function.

Let $A_i = A(r(t))$, $B_i = B(r(t))$, $C_i = C(r(t))$, $D_i = D(r(t))$ and $g_i(\cdot) = g(\cdot)$, for $r(t) = i \in S$. Then, system (2.1) becomes:

$$\begin{cases} \dfrac{\partial y}{\partial t} = \displaystyle\sum_{l=1}^{n} \dfrac{\partial}{\partial x_l}\Big(\alpha_l(t,x,y)\dfrac{\partial y}{\partial x_l}\Big) - A_i y + B_i f(y) + C_i f(y_{\varrho_1}) + D_i \dfrac{\partial y_{\varrho_2}}{\partial t} + J \\ \qquad\qquad + g_i(t,x,y,y_{\varrho_1},y_{\varrho_2})\dot{w}(t), \quad t \geq t_0,\ x \in G,\ i \in S, \\ y(\vartheta,x) = \psi(\vartheta,x), \quad \vartheta \in [-\varrho,0], \quad x \in G, \\ \dfrac{\partial y}{\partial \mathcal{N}} = 0, \quad t \in [0,+\infty),\ x \in \partial G. \end{cases} \tag{2.2}$$

We assume that the following hypotheses are held.

(H_1) For f_k, there are scalars l_k and $\bar{l}_k$ $(k = 1,\cdots,m)$, s.t.,

$$l_k \leq \tfrac{f_k(\phi_1)-f_k(\phi_2)}{\phi_1-\phi_2} \leq \bar{l}_k. \tag{2.3}$$

For any $\phi_1,\phi_2 \in \mathbb{R}$, $\phi_1 \neq \phi_2$, in which l_k and $\bar{l}_k$ can be zero, positive or negative. We suppose that

$$L = \mathrm{diag}(l_1,\cdots,l_m), \quad \bar{L} = \mathrm{diag}(\bar{l}_1,\cdots,\bar{l}_m). \tag{2.4}$$

(H_2) For $y_1,\ y_2,\ y_3 \in \mathbb{R}^m$, $t \in \mathbb{R}^+$, there exist matrices $M_{1i} > 0$, $M_{2i} > 0$, $M_{3i} > 0$, $i \in S$, s.t.,

$$\mathrm{trace}\Big(g_i^{\mathrm{T}}(t,x,y_1,y_2,y_3)g_i(t,x,y_1,y_2,y_3)\Big) \leq y_1^{\mathrm{T}}M_{1i}y_1 + y_2^{\mathrm{T}}M_{2i}y_2 + y_3^{\mathrm{T}}M_{3i}y_3. \tag{2.5}$$

Let $y^* = (y_1^*,\cdots,y_m^*)^{\mathrm{T}} \in \mathbb{R}^m$ be the equilibrium point of system (2.2), and we remove the equilibrium y^* to the original point by conversion $u = y - y^*$. Then, (2.2) becomes

$$\begin{cases} \dfrac{\partial u}{\partial t} = \displaystyle\sum_{l=1}^{n} \dfrac{\partial}{\partial x_l}\Big(\alpha_l(t,x,u)\dfrac{\partial u}{\partial x_l}\Big) - A_i p + B_i \bar{f}(u) + C_i \bar{f}(u_{\varrho_1}) + D_i \dfrac{\partial u_{\varrho_2}}{\partial t} \\ \qquad\qquad + g_i(t,x,u,u_{\varrho_1},u_{\varrho_2})\dot{w}(t), \quad t \geq t_0,\ x \in G,\ i \in S, \\ u(\vartheta,x) = \varpi(\vartheta,x), \quad \vartheta \in [-\varrho,0], \quad x \in G, \\ \dfrac{\partial u}{\partial \mathcal{N}} = 0, \quad t \geq t_0,\ x \in \partial G, \end{cases} \tag{2.6}$$

where $\bar{f}(u) = f(u+y^*) - f(y^*)$, and satisfies the condition (H_1). Let $f(0) = 0$, $g_i(t,x,0,0,0) = 0$, in which (2.6) acknowledges a trivial solution $u(t,x) = 0$. We denote $p(t,x;\varpi)$ is the state trajectory with initial condition $\varpi \in C$. Thus, system (2.6) has a trivial solution $u(t,x;0) = 0$ with initial condition $\varpi(\vartheta,x) = 0$.

Definition 2.1. ([40]) For any $\varpi \in C$, $\exists \beta > 0$, $\rho > 0$, s.t.,

$$\mathrm{E}\{\|u(t,x;\varpi,i_0)\|^2\} < \rho \mathrm{e}^{-\beta(t-t_0)} \sup_{-\varrho \le \vartheta \le 0} \mathrm{E}\|\varpi(\vartheta,x)\|^2$$

for $t \ge 0$, $i_0 \in S$. Then, the trivial solution $u(t,x) = 0$ of (2.6) is exponential stable in mean square.

3. Stability of Markovian switching time-varying delayed RNSNNs

In this section, we gain the following conclusions.

Theorem 3.1. *Assume that (H_1) and (H_2) hold. The trivial solution of (2.6) is exponential stable in mean square, if there are $\lambda_i > 0$, $\beta > 0$, $\varepsilon > 0$, positive definite matrices Q_i $(i \in S)$ and P_ν $(\nu = 1, \cdots, 6)$ such that*

$$Q_i \le \lambda_i I, \tag{3.1}$$

$$\Theta_i = \begin{bmatrix} \Theta_{i1,1} & \Theta_{i1,2} & \Theta_{i1,3} & \Theta_{i1,4} & \Theta_{i1,5} & 0 & \Theta_{i1,7} & \Theta_{i1,8} & \Theta_{i1,9} & 0 & 0 & 0 \\ * & \Theta_{i2,2} & \Theta_{i2,3} & 0 & \Theta_{i2,5} & 0 & 0 & \Theta_{i2,8} & \Theta_{i2,9} & 0 & 0 & 0 \\ * & * & \Theta_{i3,3} & 0 & 0 & 0 & 0 & \Theta_{i3,8} & \Theta_{i3,9} & 0 & 0 & 0 \\ * & * & & \Theta_{i4,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Theta_{i5,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Theta_{i6,6} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Theta_{i7,7} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Theta_{i8,8} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \Theta_{i9,9} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Theta_{i10,10} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \Theta_{i11,11} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & \Theta_{i12,12} \end{bmatrix} < 0, \tag{3.2}$$

where

$$\begin{aligned} \Theta_{i1,1} =& -Q_iA_i - A_i^{\mathrm{T}}Q_i + \beta Q_i - 2Q_i\bar{\alpha} + \lambda_i M_{1i} + \sum_{j=1}^{N} \pi_{ij} Q_j \\ &+ P_1 + P_2 + \frac{\varrho_1 - \varrho_0}{\beta}(\mathrm{e}^{\beta\varrho_1} - \mathrm{e}^{\beta\varrho_0})P_4 - 2LO_i\bar{L} + 2S_1 + 2T_1, \end{aligned}$$

$$\begin{aligned} \Theta_{i1,2} =& -S_1 + S_2^{\mathrm{T}} + T_2^{\mathrm{T}},\ \Theta_{i1,3} = -T_1 + S_3^{\mathrm{T}} + T_3^{\mathrm{T}}, \\ \Theta_{i1,4} =& Q_iB_i + (L + \bar{L})O_i, \quad \Theta_{i1,5} = Q_iC_i, \quad \Theta_{i1,7} = Q_iD_i, \quad \Theta_{i1,8} = -S_1 + S_4^{\mathrm{T}}, \\ \Theta_{i1,9} =& -T_1 + T_4^{\mathrm{T}}, \quad \Theta_{i2,2} = \lambda_i M_{2i} - (1-\kappa_1)\mathrm{e}^{-\beta\varrho_1}P_1 - 2LR_i\bar{L} - 2S_2, \\ \Theta_{i2,3} =& -T_2 - S_3^{\mathrm{T}},\ \Theta_{i2,5} = (L + \bar{L})R_i,\ \Theta_{i2,8} = -S_2 - S_4^{\mathrm{T}},\ \Theta_{i2,9} = -T_2, \\ \Theta_{i3,3} =& \lambda_i M_{3i} - (1-\kappa_2)\mathrm{e}^{-\beta\varrho_2}P_2 - 2T_3,\ \Theta_{i3,8} = -S_3,\ \Theta_{i3,9} = -T_3 - T_4^{\mathrm{T}}, \\ \Theta_{i4,4} =& \frac{\varrho}{\beta}(\mathrm{e}^{\beta\varrho} - 1)P_3 - 2O_i,\ \Theta_{i5,5} = -2R_i,\ \Theta_{i6,6} = P_5 + \frac{\varrho_2}{\beta}(\mathrm{e}^{\beta\varrho_2} - 1)P_6, \\ \Theta_{i7,7} =& -(1-\kappa_2)\mathrm{e}^{-\beta\varrho_2}P_5,\ \Theta_{i8,8} = -2S_4,\ \Theta_{i9,9} = -2T_4, \\ \Theta_{i10,10} =& -P_3,\ \Theta_{i11,11} = -P_4,\ \Theta_{i12,12} = -P_6. \end{aligned}$$

Proof. Consider the LK functional

$$V_i(t,u) = V_{1i}(t,u) + V_{2i}(t,u) + V_{3i}(t,u) + V_{4i}(t,u), \quad i \in S, \tag{3.3}$$

where

$$\begin{aligned}
V_{1i}(t,u) =& \mathrm{e}^{\beta t}\int_G u^{\mathrm{T}} Q_i u dx,\\
V_{2i}(t,u) =& \int_G \int_{t-\varrho_1(t)}^{t} \mathrm{e}^{\beta s} u^{\mathrm{T}}(s,x) P_1 u(s,x) ds dx + \int_G \int_{t-\varrho_2(t)}^{t} \mathrm{e}^{\beta s} u^{\mathrm{T}}(s,x) P_2 u(s,x) ds dx,\\
V_{3i}(t,u) =& \varrho \int_G \int_{-\varrho}^{0} \int_{t+\gamma}^{t} \mathrm{e}^{\beta(s-\gamma)} \bar{f}^{\mathrm{T}}(u(s,x)) P_3 \bar{f}(u(s,x)) ds d\gamma dx\\
&+ (\varrho_1 - \varrho_0) \int_G \int_{-\varrho_1}^{-\varrho_0} \int_{t+\gamma}^{t} \mathrm{e}^{\beta(s-\gamma)} u^{\mathrm{T}}(s,x) P_4 u(s,x) ds d\gamma dx,\\
V_{4i}(t,u) =& \int_G \int_{t-\varrho_2(t)}^{t} \mathrm{e}^{\beta s} (\frac{\partial u(s,x)}{\partial s})^{\mathrm{T}} P_5 (\frac{\partial u(s,x)}{\partial s}) ds dx\\
&+ \varrho_2 \int_G \int_{-\varrho_2}^{0} \int_{t+\gamma}^{t} \mathrm{e}^{\beta(s-\gamma)} (\frac{\partial u(s,x)}{\partial s})^{\mathrm{T}} P_6 (\frac{\partial u(s,x)}{\partial s}) ds d\gamma dx.
\end{aligned}$$

Let $\mathcal{L}$ be the weak infinitesimal operator of random process $\{u(t,x;r(t)), t \geq 0\}$, we obtain

$$\mathcal{L}V_i = \mathcal{L}V_{1i} + \mathcal{L}V_{2i} + \mathcal{L}V_{3i} + \mathcal{L}V_{4i}, \quad i \in S, \tag{3.4}$$

where

$$\begin{aligned}
\mathcal{L}V_{1i}(t,u) =& \beta \mathrm{e}^{\beta t} \int_G u^{\mathrm{T}} Q_i u dx + 2\mathrm{e}^{\beta t} \int_G u^{\mathrm{T}} Q_i \Big[\sum_{l=1}^{n} \frac{\partial}{\partial x_l} \Big(\alpha_l(t,x,u) \frac{\partial u}{\partial x_l} \Big)\\
&- A_i u + B_i \bar{f}(u) + C_i \bar{f}(u_{\varrho_1}) + D_i \frac{\partial u_{\varrho_2}}{\partial t} \Big] dx\\
&+ \mathrm{e}^{\beta t} \int_G \mathrm{trace}\Big[g_i^{\mathrm{T}}(\cdot) Q_i g_i(\cdot) \Big] dx + \mathrm{e}^{\beta t} \int_G u^{\mathrm{T}} \Big(\sum_{j=1}^{N} \pi_{ij} Q_j \Big) u dx,
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
\mathcal{L}V_{2i}(t,u) =& \mathrm{e}^{\beta t} \int_G \Big[u^{\mathrm{T}} P_1 u - (1 - \dot{\varrho}_1(t)) \mathrm{e}^{-\beta \varrho_1(t)} u_{\varrho_1}^{\mathrm{T}} P_1 u_{\varrho_1} \Big] dx\\
&+ \mathrm{e}^{\beta t} \int_G \Big[u^{\mathrm{T}} P_2 u - (1 - \dot{\varrho}_2(t)) \mathrm{e}^{-\beta \varrho_2(t)} u_{\varrho_2}^{\mathrm{T}} P_2 u_{\varrho_2} \Big] dx,\\
\mathcal{L}V_{3i}(t,u) =& \varrho \int_G \int_{-\varrho}^{0} \mathrm{e}^{\beta(t-\gamma)} \bar{f}^{\mathrm{T}}(u(t,x)) P_3 \bar{f}(u(t,x)) d\gamma dx\\
&- \varrho \int_G \int_{-\varrho}^{0} \mathrm{e}^{\beta t} \bar{f}^{\mathrm{T}}(u(t+\gamma,x)) P_3 \bar{f}(u(t+\gamma,x)) d\gamma dx\\
&+ (\varrho_1 - \varrho_0) \int_G \int_{-\varrho_1}^{-\varrho_0} \mathrm{e}^{\beta(t-\gamma)} u^{\mathrm{T}}(t,x) P_4 u(t,x) d\gamma dx
\end{aligned} \tag{3.6}$$

$$
\begin{aligned}
&-(\varrho_1-\varrho_0)\int_G\int_{-\varrho_1}^{-\varrho_0}\mathrm{e}^{\beta t}u^{\mathrm{T}}(t+\gamma,x)P_4u(t+\gamma,x)d\gamma dx\\
\le&\frac{\varrho}{\beta}\int_G(\mathrm{e}^{\beta\varrho}-1)\mathrm{e}^{\beta t}\bar f^{\mathrm{T}}(u)P_3\bar f(u)dx-\varrho\mathrm{e}^{\beta t}\int_G\int_{t-\varrho}^{t}\bar f^{\mathrm{T}}(u(s,x))P_3\bar f(u(s,x))dsdx\\
&+\frac{\varrho_1-\varrho_0}{\beta}\int_G(\mathrm{e}^{\beta\varrho_1}-\mathrm{e}^{\beta\varrho_0})u^{\mathrm{T}}P_4udx-(\varrho_1-\varrho_0)\mathrm{e}^{\beta t}\int_G\int_{t-\varrho_1}^{t-\varrho_0}u^{\mathrm{T}}(s,x)P_4u(s,x)dsdx\\
\le&\frac{\varrho}{\beta}\int_G(\mathrm{e}^{\beta\varrho}-1)\mathrm{e}^{\beta t}\bar f^{\mathrm{T}}(u)P_3\bar f(u)dx-\varrho_1(t)\mathrm{e}^{\beta t}\int_G\int_{t-\varrho_1(t)}^{t}\bar f^{\mathrm{T}}(u(s,x))P_3\bar f(u(s,x))dsdx\\
&+\frac{\varrho_1-\varrho_0}{\beta}\int_G(\mathrm{e}^{\beta\varrho_1}-\mathrm{e}^{\beta\varrho_0})u^{\mathrm{T}}P_4udx-(\varrho_1-\varrho_0)\mathrm{e}^{\beta t}\int_G\int_{t-\varrho_1}^{t-\varrho_0}u^{\mathrm{T}}(s,x)P_4u(s,x)dsdx,
\end{aligned}
\tag{3.7}
$$

$$
\begin{aligned}
\mathcal{L}V_{4i}(t,u)=&\mathrm{e}^{\beta t}\int_G\Big[(\frac{\partial u}{\partial t})^{\mathrm{T}}P_5(\frac{\partial u}{\partial t})-(1-\dot\varrho_2(t))\mathrm{e}^{-\beta\varrho_2(t)}(\frac{\partial u_{\varrho_2}}{\partial t})^{\mathrm{T}}P_5(\frac{\partial u_{\varrho_2}}{\partial t})\Big]dx\\
&+\int_G\int_{-\varrho_2}^{0}\varrho_2\Big[\mathrm{e}^{\beta(t-\gamma)}(\frac{\partial u(t,x)}{\partial t})^{\mathrm{T}}P_6(\frac{\partial u(t,x)}{\partial t})\\
&-\mathrm{e}^{\beta t}(\frac{\partial u(t+\gamma,x)}{\partial t})^{\mathrm{T}}P_6(\frac{\partial u(t+\gamma,x)}{\partial t})\Big]d\gamma dx.
\end{aligned}
\tag{3.8}
$$

By Green's formula, boundary condition and Poincarè inequality, we gain

$$
\begin{aligned}
\int_G u^{\mathrm{T}}\sum_{l=1}^{n}\frac{\partial}{\partial x_l}\Big(\alpha_l\frac{\partial u}{\partial x_l}\Big)dx=&\int_G\sum_{k=1}^{m}\sum_{l=1}^{n}u_k^{\mathrm{T}}\frac{\partial}{\partial x_l}\Big(\alpha_{kl}\frac{\partial u_k}{\partial x_l}\Big)dx\\
=&-\sum_{k=1}^{m}\sum_{l=1}^{n}\int_G\alpha_{kl}\Big(\frac{\partial u_k}{\partial x_l}\Big)^2dx\\
\le&-\sum_{k=1}^{m}\sum_{l=1}^{n}\int_G\alpha_{kl}\Big(\frac{\partial u_k}{\partial x_l}\Big)^2dx\\
\le&-\sum_{k=1}^{m}\sum_{l=1}^{n}\int_G\frac{\alpha_{kl}}{d_l^2}u_k^2dx.
\end{aligned}
\tag{3.9}
$$

Applying hypothesis (H_2), we gain that there are positive matrices $O_i=\mathrm{diag}(o_{1i},\cdots,o_{mi})$ and $R_i=\mathrm{diag}(r_{1i},\cdots,r_{mi})$ $(i\in S)$ such that

$$
\begin{aligned}
0\le&2\mathrm{e}^{\beta t}\sum_{q=1}^{m}o_{qi}\Big(\bar f_q(u_q)-l_qu_q\Big)\Big(\bar l_qu_q-\bar f_q(u_q)\Big)\\
=&2\mathrm{e}^{\beta t}\Big[u^{\mathrm{T}}(L+\bar L)O_i\bar f(u)-u^{\mathrm{T}}LO_i\bar Lu-\bar f^{\mathrm{T}}(u)O_i\bar f(u)\Big],\\
0\le&2\mathrm{e}^{\beta t}\sum_{q=1}^{m}r_{qi}\Big(\bar f_q(u_{q\varrho_1})-l_qu_{q\varrho_1}\Big)\Big(\bar l_qu_{q\varrho_1}-\bar f_q(u_{q\varrho_1})\Big)\\
=&2\mathrm{e}^{\beta t}\Big[u_{\varrho_1}^{\mathrm{T}}(L+\bar L)R_i\bar f(u_{\varrho_1})-u_{\varrho_1}^{\mathrm{T}}LR_i\bar Lu_{\varrho_1}-\bar f^{\mathrm{T}}(u_{\varrho_1})R_i\bar f(u_{\varrho_1})\Big].
\end{aligned}
\tag{3.10}
$$

On the basis of the Newton-Leibniz formula, there exist appropriate dimensions matrices S_ϵ, U_ϵ and W_ϵ $(\epsilon=1,2,3,4)$ such that

$$
0=\int_G\Big[2\Big(u^{\mathrm{T}}S_1+u_{\varrho_1}^{\mathrm{T}}S_2+u_{\varrho_2}^{\mathrm{T}}S_3
$$

$$+ (\int_{t-\varrho_1(t)}^{t} \frac{\partial u(s,x)}{\partial s} ds)^{\mathrm{T}} S_4\Big) \cdot \Big(u - u_{\varrho_1} - \int_{t-\varrho_1(t)}^{t} \frac{\partial u(s,x)}{\partial s} ds\Big)\Big] dx,$$

$$0 = \int_G \Big[2\Big(u^{\mathrm{T}} T_1 + u_{\varrho_1}^{\mathrm{T}} T_2 + u_{\varrho_2}^{\mathrm{T}} T_3 \tag{3.11}$$

$$+ (\int_{t-\varrho_2(t)}^{t} \frac{\partial u(s,x)}{\partial s} ds)^{\mathrm{T}} T_4\Big) \cdot \Big(u - u_{\varrho_2} - \int_{t-\varrho_2(t)}^{t} \frac{\partial u(s,x)}{\partial s} ds\Big)\Big] dx.$$

Then,

$$\begin{aligned}\mathcal{L}V_{1i}(t,u) \le & \mathrm{e}^{\beta t} \int_G \Big\{ u^{\mathrm{T}} \Big[\beta Q_i - 2Q_i \bar{\alpha} - Q_i A_i - A_i^{\mathrm{T}} Q_i + \lambda_i M_{1i} + \sum_{j=1}^{N} \pi_{ij} Q_j\Big] u \\ & + 2u^{\mathrm{T}} Q_i B_i \bar{f}(u) + 2u^{\mathrm{T}} Q_i C_i \bar{f}(u_{\varrho_1}) + \lambda_i u_{\varrho_1}^{\mathrm{T}} M_{2i} u_{\varrho_1} + \lambda_i u_{\varrho_2}^{\mathrm{T}} M_{3i} u_{\varrho_2} \\ & + 2u^{\mathrm{T}} Q_i D_i \frac{\partial u_{\varrho_2}}{\partial t} \Big\} dx,\end{aligned} \tag{3.12}$$

where $\bar{\alpha} = \mathrm{diag}\Big(\sum\limits_{l=1}^{n} \frac{\alpha_{1l}}{d_l^2}, \cdots, \sum\limits_{l=1}^{n} \frac{\alpha_{ml}}{d_l^2}\Big)$. By (3.6), we obtain

$$\mathcal{L}V_{2i}(t,u) \le \mathrm{e}^{\beta t} \int_G \Big[u^{\mathrm{T}}(P_1 + P_2)u - (1-\kappa_1)\mathrm{e}^{-\beta\varrho_1} u_{\varrho_1}^{\mathrm{T}} P_1 u_{\varrho_1} - (1-\kappa_2)\mathrm{e}^{-\beta\varrho_2} u_{\varrho_2}^{\mathrm{T}} P_2 u_{\varrho_2}\Big] dx. \tag{3.13}$$

Noting that $\varrho_1(t) > 0$ and $\varrho_1 > \varrho_0$, by Jensen integral inequality (see [12]), we gain

$$\begin{aligned}\mathcal{L}V_{3i}(t,u) \le & \mathrm{e}^{\beta t} \int_G \Big\{ \frac{\varrho}{\beta}(\mathrm{e}^{\beta\varrho} - 1)\bar{f}^{\mathrm{T}}(u) P_3 \bar{f}(u) \\ & - (\int_{t-\varrho_1(t)}^{t} \bar{f}(u(s,x))ds)^{\mathrm{T}} P_3 (\int_{t-\varrho_1(t)}^{t} \bar{f}(u(s,x))ds) \\ & + \frac{\varrho_1 - \varrho_0}{\beta}(\mathrm{e}^{\beta\varrho_1} - \mathrm{e}^{\beta\varrho_0}) u^{\mathrm{T}} P_4 u - (\int_{t-\varrho_1}^{t-\varrho_0} u(s,x)ds)^{\mathrm{T}} P_4 (\int_{t-\varrho_1}^{t-\varrho_0} u(s,x)ds) \Big\} dx.\end{aligned} \tag{3.14}$$

If $\varrho_1(t) = 0$, and $\varrho_1 = \varrho_0$, (3.14) still holds, i.e.,

$$\begin{aligned} & \varrho_2(t) \int_{t-\varrho}^{t} \bar{f}^{\mathrm{T}}(u(s,x)) P_3 \bar{f}(u(s,x))ds = (\int_{t-\varrho_1(t)}^{t} \bar{f}(u(s,x))ds)^{\mathrm{T}} P_3 (\int_{t-\varrho_1(t)}^{t} \bar{f}(u(s,x))ds) = 0, \\ & (\varrho_1 - \varrho_0) \int_{t-\varrho_1}^{t-\varrho_0} u^{\mathrm{T}}(s,x) P_4 u(s,x)ds = (\int_{t-\varrho_1}^{t-\varrho_0} u(s,x)ds)^{\mathrm{T}} P_4 (\int_{t-\varrho_1}^{t-\varrho_0} u(s,x)ds) = 0.\end{aligned} \tag{3.15}$$

By (3.8) and Jensen integral inequality, we have

$$\begin{aligned}\mathcal{L}V_{4i}(t,u) \le & \mathrm{e}^{\beta t} \int_G \Big[(\frac{\partial u}{\partial t})^{\mathrm{T}} (P_5 + \frac{\varrho_2}{\beta}(\mathrm{e}^{\beta\varrho_2} - 1)P_6)(\frac{\partial u}{\partial t}) - (1-\kappa_2)\mathrm{e}^{-\beta\varrho_2} (\frac{\partial u_{\varrho_2}}{\partial t})^{\mathrm{T}} P_5 (\frac{\partial u_{\varrho_2}}{\partial t}) \\ & - \varrho_2 \int_{t-\varrho_2}^{t} (\frac{\partial u(s,x)}{\partial s})^{\mathrm{T}} P_6 (\frac{\partial u(s,x)}{\partial s}) ds\Big] dx \\ \le & \mathrm{e}^{\beta t} \int_G \Big[(\frac{\partial u}{\partial t})^{\mathrm{T}} \Big(P_5 + \frac{\varrho_2}{\beta}(\mathrm{e}^{\beta\varrho_2} - 1)P_6\Big)(\frac{\partial u}{\partial t}) - (1-\kappa_2)\mathrm{e}^{-\beta\varrho_2} (\frac{\partial u_{\varrho_2}}{\partial t})^{\mathrm{T}} P_5 (\frac{\partial u_{\varrho_2}}{\partial t}) \\ & - (\int_{t-\varrho_2}^{t} \frac{\partial u(s,x)}{\partial s} ds)^{\mathrm{T}} P_6 (\int_{t-\varrho_2}^{t} \frac{\partial u(s,x)}{\partial s} ds)\Big] dx.\end{aligned} \tag{3.16}$$

Thus, by (3.10)-(3.16), we have

$$
\begin{aligned}
&\mathcal{L}V_i(t,u)\\
\le\ & \mathrm{e}^{\beta t}\int_G\Big\{u^{\mathrm T}\Big[-Q_iA_i-A_i^{\mathrm T}Q_i+\beta Q_i-2Q_i\bar\alpha+\lambda_iM_{1i}+\sum_{j=1}^{N}\pi_{ij}Q_j+P_1+P_2\\
&+\frac{\varrho_1-\varrho_0}{\beta}(\mathrm{e}^{\beta\varrho_1}-\mathrm{e}^{\beta\varrho_0})P_4-2LO_i\bar L+2S_1+2T_1\Big]u-2u^{\mathrm T}S_1u_{\varrho_1}-2u^{\mathrm T}T_1u_{\varrho_2}\\
&-2u^{\mathrm T}S_1(\int_{t-\varrho_1(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)-2u^{\mathrm T}T_1(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)+2u_{\varrho_1}^{\mathrm T}(S_2+T_2)u\\
&+2u_{\varrho_2}^{\mathrm T}(S_3+T_3)u-2u_{\varrho_1}^{\mathrm T}S_2u_{\varrho_1}-2u_{\varrho_2}^{\mathrm T}T_3u_{\varrho_2}-2u_{\varrho_1}^{\mathrm T}T_2u_{\varrho_2}-2u_{\varrho_2}^{\mathrm T}S_3u_{\varrho_1}\\
&-2u_{\varrho_1}^{\mathrm T}S_2(\int_{t-\varrho_1(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)-2u_{\varrho_2}^{\mathrm T}S_3(\int_{t-\varrho_1(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)\\
&-2u_{\varrho_1}^{\mathrm T}T_2(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)-2u_{\varrho_2}^{\mathrm T}T_3(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)\\
&+2(\int_{t-\varrho_1(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm T}S_4u+2(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm T}T_4u\\
&-2(\int_{t-\varrho_1(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm T}S_4u_{\varrho_1}-2(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm T}T_4u_{\varrho_2}\\
&-2(\int_{t-\varrho_1(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm T}S_4(\int_{t-\varrho_1(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)\\
&-2(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm T}T_4(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)\\
&+2u^{\mathrm T}\Big[Q_iB_i+(L+\bar L)O_i\Big]\bar f(u)+2u^{\mathrm T}Q_iC_i\bar f(u_{\varrho_1})+2u^{\mathrm T}Q_iD_i\frac{\partial u_{\varrho_2}}{\partial t}\\
&+\bar f^{\mathrm T}(u)\Big[\frac{\varrho}{\beta}(\mathrm{e}^{\beta\varrho}-1)P_3-2O_i\Big]\bar f(u)+u_{\varrho_1}^{\mathrm T}\Big[\lambda_iM_{2i}-(1-\kappa_1)\mathrm{e}^{-\beta\varrho_1}P_1-2LR_i\bar L\Big]u_{\varrho_1}\\
&+2u_{\varrho_1}^{\mathrm T}(L+\bar L)R_i\bar f(u_{\varrho_1})-2\bar f^{\mathrm T}(u_{\varrho_1})R_i\bar f(u_{\varrho_1})+u_{\varrho_2}^{\mathrm T}\Big[\lambda_iM_{3i}-(1-\kappa_2)\mathrm{e}^{-\beta\varrho_2}P_2\Big]u_{\varrho_2}\\
&-(\int_{t-\varrho_1(t)}^{t}\bar f(u(s,x))ds)^{\mathrm T}P_3(\int_{t-\varrho_1(t)}^{t}\bar f(u(s,x))ds)\\
&-(\int_{t-\varrho_1}^{t-\varrho_0}u(s,x)ds)^{\mathrm T}P_4(\int_{t-\varrho_1}^{t-\varrho_0}u(s,x)ds)\\
&-(\int_{t-\varrho_2}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm T}P_6(\int_{t-\varrho_2}^{t}\frac{\partial u(s,x)}{\partial s}ds)+(\frac{\partial u}{\partial t})^{\mathrm T}\Big[P_5+\frac{\varrho_2}{\beta}(\mathrm{e}^{\beta\varrho_2}-1)P_6\Big](\frac{\partial u}{\partial t})\\
&-(1-\kappa_2)\mathrm{e}^{-\beta\varrho_2}(\frac{\partial u_{\varrho_2}}{\partial t})^{\mathrm T}P_5(\frac{\partial u_{\varrho_2}}{\partial t})\Big\}dx\\
=\ & \mathrm{e}^{\beta t}\int_G\zeta^{\mathrm T}(t)\Theta_i\zeta(t)dx,\quad \forall t\ge t_0, i\in S,
\end{aligned}
\tag{3.17}
$$

in which

$$
\begin{aligned}
\zeta^T(t)=\Big[&u^{\mathrm T},u_{\varrho_1}^{\mathrm T},u_{\varrho_2}^{\mathrm T},\bar f^{\mathrm T}(u),\bar f^{\mathrm T}(u_{\varrho_1}),(\frac{\partial u}{\partial t})^{\mathrm T},(\frac{\partial u_{\varrho_2}}{\partial t})^{\mathrm T},(\int_{t-\varrho_1(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm T},\\
&(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm T},(\int_{t-\varrho_1(t)}^{t}\bar f(u(s,x))ds)^{\mathrm T},(\int_{t-\varrho_1}^{t-\varrho_0}u(s,x)ds)^{\mathrm T},\\
&(\int_{t-\varrho_2}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm T}\Big].
\end{aligned}
$$

By (3.2), $\mathcal{L}V_i(t,u)<0$ for $t\geq t_0$ $(i\in S)$. Thus, it is easily achieved that

$$\mathrm{E}\mathcal{L}V_i(t,u)\leq 0,\quad t\geq t_0,\ i\in S. \tag{3.18}$$

Through (3.18), we obtain

$$\mathrm{E}V_i(t,u)<\mathrm{E}V_i(t_0,u(t_0,x)),\quad t\geq t_0,\ i\in S. \tag{3.19}$$

Noting that

$$\mathrm{E}V_i(t,u)\geq \lambda_{\min}(Q_i)\mathrm{e}^{\beta t}\mathrm{E}\|u\|^2, \tag{3.20}$$

and

$$\begin{aligned}
\mathrm{E}V_i(t_0,u(t_0,x)) = {} & \mathrm{e}^{\beta t_0}\int_G u^{\mathrm{T}}(t_0,x)Q_i u(t_0,x)dx\\
&+\int_G\int_{t_0-\varrho_1(t_0)}^{t_0}\mathrm{e}^{\beta s}u^{\mathrm{T}}(s,x)P_1u(s,x)dsdx\\
&+\int_G\int_{t_0-\varrho_2(t_0)}^{t_0}\mathrm{e}^{\beta s}u^{\mathrm{T}}(s,x)P_2u(s,x)dsdx\\
&+\tau\int_G\int_{-\varrho}^{0}\int_{t_0+\gamma}^{t_0}\mathrm{e}^{\beta(s-\gamma)}\bar{f}^{\mathrm{T}}(u(s,x))P_3\bar{f}(u(s,x))dsd\gamma dx\\
&+(\varrho_1-\varrho_0)\int_G\int_{-\varrho_1}^{-\varrho_0}\int_{t_0+\gamma}^{t_0}\mathrm{e}^{\beta(s-\gamma)}u^{\mathrm{T}}(s,x)P_4u(s,x)dsd\gamma dx\\
&+\int_G\int_{t_0-\varrho_2(t_0)}^{t_0}\mathrm{e}^{\beta s}\Big(\frac{\partial u(s,x)}{\partial s}\Big)^{\mathrm{T}}P_5\Big(\frac{\partial u(s,x)}{\partial s}\Big)dsdx\\
&+\varrho_2\int_G\int_{-\varrho_2}^{0}\int_{t_0+\gamma}^{t_0}\mathrm{e}^{\beta(s-\gamma)}\Big(\frac{\partial u(s,x)}{\partial s}\Big)^{\mathrm{T}}P_6\Big(\frac{\partial u(s,x)}{\partial s}\Big)dsd\gamma dx
\end{aligned}$$

$$\begin{aligned}
\leq{} & \mathrm{e}^{\beta t_0}\Big\{\lambda_{\max}(Q_i)+\lambda_{\max}(P_1)\frac{1-\mathrm{e}^{-\beta\varrho_1}}{\beta}+\lambda_{\max}(P_2)\frac{1-\mathrm{e}^{-\beta\varrho_2}}{\beta}\\
&+\lambda_{\max}(\bar{L}P_3\bar{L})\Big[\frac{\varrho}{\beta^2}(\mathrm{e}^{\beta\varrho_1}-1)-\frac{\varrho^2}{\beta}\Big]+\lambda_{\max}(P_4)\Big[\frac{\varrho_1-\varrho_0}{\beta^2}(\mathrm{e}^{\beta\varrho_1}-\mathrm{e}^{\beta\varrho_0})-\frac{(\varrho_1-\varrho_0)^2}{\beta}\Big]\\
&+\lambda_{\max}(P_5)\frac{1-\mathrm{e}^{-\beta\varrho_2}}{\beta}+\lambda_{\max}(P_6)\Big[\frac{\varrho_2}{\beta^2}(\mathrm{e}^{\beta\varrho_2}-1)-\frac{\varrho_2^2}{\beta}\Big]\Big\}\sup_{-\varrho\leq\vartheta\leq 0}\mathrm{E}\|\varpi(\vartheta,x)\|^2,
\end{aligned} \tag{3.21}$$

Then, from (3.20) and (3.21), we gain

$$\mathrm{E}\|u\|^2\leq \mathrm{e}^{-\beta(t-t_0)}\rho\sup_{-\varrho\leq\vartheta\leq 0}\mathrm{E}\|\varpi(\vartheta,x)\|^2,\quad t\geq t_0, \tag{3.22}$$

where

$$\begin{aligned}
\rho ={} & \frac{\bar{\rho}}{\lambda_{\min}(Q_i)},\\
\bar{\rho} ={} & \lambda_{\max}(Q_i)+\lambda_{\max}(P_1)\frac{1-\mathrm{e}^{-\beta\varrho_1}}{\beta}+\lambda_{\max}(P_2)\frac{1-\mathrm{e}^{-\beta\varrho_2}}{\beta}\\
&+\lambda_{\max}(\bar{L}P_3\bar{L})\Big[\frac{\varrho}{\beta^2}(\mathrm{e}^{\beta\varrho_1}-1)-\frac{\varrho^2}{\beta}\Big]\\
&+\lambda_{\max}(P_4)\Big[\frac{\varrho_1-\varrho_0}{\beta^2}(\mathrm{e}^{\beta\varrho_1}-\mathrm{e}^{\beta\varrho_0})-\frac{(\varrho_1-\varrho_0)^2}{\beta}\Big]+\lambda_{\max}(P_5)\frac{1-\mathrm{e}^{-\beta\varrho_2}}{\beta}\\
&+\lambda_{\max}(P_6)\Big[\frac{\varrho_2}{\beta^2}(\mathrm{e}^{\beta\varrho_2}-1)-\frac{\varrho_2^2}{\beta}\Big],\quad i\in S.
\end{aligned}$$

Thus, from (3.22), we implies that $u(t,x)=0$ of (2.6) is exponential stable in mean square. There ends the proof. $\square$

In the next step, we will consider the time-varying delayed neutral-type reaction-diffusion systems without Markovian switching and stochastic perturbations:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \displaystyle\sum_{l=1}^{n} \frac{\partial}{\partial x_l}\Big(\alpha_l(t,x,u)\frac{\partial u}{\partial x_l}\Big) - Au + B\bar{f}(u) + C\bar{f}(u_{\varrho_1}) + D\frac{\partial u_{\varrho_2}}{\partial t}, \quad t \geq t_0,\ x \in G, \\ u(\vartheta,x) = \varpi(\vartheta,x), \quad -\varrho \leq \vartheta \leq 0, \quad x \in G, \\ \dfrac{\partial u}{\partial \mathcal{N}} = 0, \quad t \geq t_0,\ x \in \partial G. \end{cases} \tag{3.23}$$

For system (3.23), we present the following theorem.

Theorem 3.2. *If assumption (H_1) holds. $u(t,x)=0$ of (3.23) is exponentially stable; if $\beta>0$, positive definite matrices Q, and P_ν $(\nu=1,2,3,4,5)$, such that*

$$\Xi = \begin{bmatrix} \Xi_{1,1} & \Xi_{1,2} & \Xi_{1,3} & \Xi_{1,4} & \Xi_{1,5} & 0 & \Xi_{1,7} & \Xi_{1,8} & \Xi_{1,9} & 0 & 0 \\ * & \Xi_{2,2} & \Xi_{2,3} & 0 & \Xi_{2,5} & 0 & 0 & \Xi_{2,8} & \Xi_{2,9} & 0 & 0 \\ * & * & \Xi_{3,3} & 0 & 0 & 0 & 0 & \Xi_{3,8} & \Xi_{3,9} & 0 & 0 \\ * & * & & \Xi_{4,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Xi_{5,5} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Xi_{6,6} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Xi_{7,7} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Xi_{8,8} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \Xi_{9,9} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Xi_{10,10} & 0 \\ * & * & * & * & * & * & * & * & * & * & \Xi_{11,11} \end{bmatrix} < 0, \tag{3.24}$$

where

$$\begin{aligned} \Xi_{1,1} =& -QA - A^{\mathrm{T}}Q + \beta Q - 2Q\bar{\alpha} + P_1 + P_2 + \frac{\varrho_1 - \varrho_0}{\beta}(\mathrm{e}^{\beta\varrho_1} - \mathrm{e}^{\beta\varrho_0})P_3 \\ & - 2LO\bar{L} + 2S_1 + 2T_1, \\ \Xi_{1,2} =& - S_1 + S_2^{\mathrm{T}} + T_2^{\mathrm{T}},\ \Xi_{1,3} = -T_1 + S_3^{\mathrm{T}} + T_3^{\mathrm{T}},\ \Xi_{1,4} = QB + (L+\bar{L})O, \\ \Xi_{1,5} =& QC, \quad \Theta_{1,7} = QD, \quad \Xi_{1,8} = -S_1 + S_4^{\mathrm{T}},\ \Xi_{1,9} = -T_1 + T_4^{\mathrm{T}}, \\ \Xi_{2,2} =& -(1-\kappa_1)\mathrm{e}^{-\beta\varrho_1}P_1 - 2LR\bar{L} - 2S_2,\ \Xi_{2,3} = -T_2 - S_3^{\mathrm{T}},\ \Xi_{2,5} = (L+\bar{L})R, \\ \Xi_{2,8} =& - S_2 - S_4^{\mathrm{T}},\ \Xi_{2,9} = -T_2,\ \Xi_{3,3} = -(1-\kappa_2)\mathrm{e}^{-\beta\varrho_2}P_2 - 2T_3,\ \Xi_{3,8} = -S_3, \\ \Xi_{3,9} =& - T_3 - T_4^{\mathrm{T}},\ \Xi_{4,4} = -2O,\ \Xi_{5,5} = -2R,\ \Xi_{6,6} = P_4 + \frac{\varrho_2}{\beta}(\mathrm{e}^{\beta\varrho_2} - 1)P_5, \\ \Xi_{7,7} =& -(1-\kappa_2)\mathrm{e}^{-\beta\varrho_2}P_4,\ \Xi_{8,8} = -2S_4,\ \Xi_{9,9} = -2T_4,\ \Xi_{10,10} = -P_3,\ \Xi_{11,11} = -P_5. \end{aligned}$$

Proof. Consider the LK functional

$$
\begin{aligned}
V(t,u) =& e^{\beta t}\int_G u^{\mathrm{T}}Qudx + \int_G\int_{t-\varrho_1(t)}^{t} e^{\beta s}u^{\mathrm{T}}(s,x)P_1u(s,x)dsdx \\
&+ \int_G\int_{t-\varrho_2(t)}^{t} e^{\beta s}u^{\mathrm{T}}(s,x)P_2u(s,x)dsdx \\
&+ (\varrho_1-\varrho_0)\int_G\int_{-\varrho_1}^{-\varrho_0}\int_{t+\gamma}^{t} e^{\beta(s-\gamma)}u^{\mathrm{T}}(s,x)P_3u(s,x)dsd\gamma dx \\
&+ \int_G\int_{t-\varrho_2(t)}^{t} e^{\beta s}(\frac{\partial u(s,x)}{\partial s})^{\mathrm{T}}P_4(\frac{\partial u(s,x)}{\partial s})dsdx \\
&+ \varrho_2\int_G\int_{-\varrho_2}^{0}\int_{t+\gamma}^{t} e^{\beta(s-\gamma)}(\frac{\partial u(s,x)}{\partial s})^{\mathrm{T}}P_5(\frac{\partial u(s,x)}{\partial s})dsd\gamma dx.
\end{aligned}
\tag{3.25}
$$

Then, we acquire

$$
\begin{aligned}
\frac{dV(t,u)}{dt} =& \beta e^{\beta t}\int_G u^{\mathrm{T}}Qudx + 2e^{\beta t}\int_G u^{\mathrm{T}}Q\Big[\sum_{l=1}^{n}\frac{\partial}{\partial x_l}\Big(\alpha_l(t,x,u)\frac{\partial u}{\partial x_l}\Big) - Au + B\bar{f}(u) \\
&+ C\bar{f}(u_{\varrho_1}) + D\frac{\partial u_{\varrho_2}}{\partial t}\Big]dx + e^{\beta t}\int_G\Big[u^{\mathrm{T}}P_1u - (1-\dot{\varrho}_1(t))e^{-\beta\varrho_1(t)}u_{\varrho_1}^{\mathrm{T}}P_1u_{\varrho_1}\Big]dx \\
&+ e^{\beta t}\int_G\Big[u^{\mathrm{T}}P_2u - (1-\dot{\varrho}_2(t))e^{-\beta\varrho_2(t)}u_{\varrho_2}^{\mathrm{T}}P_2u_{\varrho_2}\Big]dx \\
&+ (\varrho_1-\varrho_0)\int_G\int_{-\varrho_1}^{-\varrho_0} e^{\beta(t-\gamma)}u^{\mathrm{T}}(t,x)P_3u(t,x)d\gamma dx \\
&- (\varrho_1-\varrho_0)\int_G\int_{-\varrho_1}^{-\varrho_0} e^{\beta t}u^{\mathrm{T}}(t+\gamma,x)P_3u(t+\gamma,x)d\gamma dx \\
&+ e^{\beta t}\int_G\Big[(\frac{\partial u}{\partial t})^{\mathrm{T}}P_4(\frac{\partial u}{\partial t}) - (1-\dot{\varrho}_2(t))e^{-\beta\varrho_2(t)}(\frac{\partial u_{\varrho_2}}{\partial t})^{\mathrm{T}}P_4(\frac{\partial u_{\varrho_2}}{\partial t})\Big]dx \\
&+ \int_G\int_{-\varrho_2}^{0}\varrho_2\Big[e^{\beta(t-\gamma)}(\frac{\partial u(t,x)}{\partial t})^{\mathrm{T}}P_5(\frac{\partial u(t,x)}{\partial t}) \\
&- e^{\beta t}(\frac{\partial u(t+\gamma,x)}{\partial t})^{\mathrm{T}}P_5(\frac{\partial u(t+\gamma,x)}{\partial t})\Big]d\gamma dx.
\end{aligned}
\tag{3.26}
$$

Noting that $\varrho_1 > \varrho_0$, by Jensen integral inequality, we derive that

$$
\begin{aligned}
&-(\varrho_1-\varrho_0)\int_{-\varrho_1}^{-\varrho_0} u^{\mathrm{T}}(t+\gamma,x)P_3u(t+\gamma,x)d\gamma \\
&\le -\Big[\int_{t-\varrho_1}^{t-\varrho_0} u(s,x)ds\Big]^{\mathrm{T}}P_3\Big[\int_{t-\varrho_1}^{t-\varrho_0} u(s,x)ds\Big].
\end{aligned}
\tag{3.27}
$$

If $\varrho_1 = \varrho_0$, (3.34) still holds, that is,

$$
(\varrho_1-\varrho_0)\int_{t-\varrho_1}^{t-\varrho_0} u^{\mathrm{T}}(s,x)P_3u(s,x)ds = \Big[\int_{t-\varrho_1}^{t-\varrho_0} u(s,x)ds\Big]^{\mathrm{T}}P_3\Big[\int_{t-\varrho_1}^{t-\varrho_0} u(s,x)ds\Big] = 0.
\tag{3.28}
$$

By Jensen integral inequality, we acquire

$$
\begin{aligned}
&- \varrho_2 \int_{t-\varrho_2}^{t} \Big(\frac{\partial u(s,x)}{\partial s}\Big)^{\mathrm{T}} P_5 \Big(\frac{\partial u(s,x)}{\partial s}\Big) ds\Big] dx \\
&\le -\Big(\int_{t-\varrho_2}^{t} \frac{\partial u(s,x)}{\partial s} ds\Big)^{\mathrm{T}} P_5 \Big(\int_{t-\varrho_2}^{t} \frac{\partial u(s,x)}{\partial s} ds\Big)\Big] dx.
\end{aligned} \tag{3.29}
$$

Applying hypothesis (H_2), we gain that there are positive matrices $O = \mathrm{diag}(o_1, \cdots, o_m)$ and $R = \mathrm{diag}(r_1, \cdots, r_m)$ such that

$$
\begin{aligned}
0 &\le 2\mathrm{e}^{\beta t} \sum_{q=1}^{m} o_q \Big(\bar{f}_q(u_q) - l_q u_q\Big)\Big(\bar{l}_q u_q - \bar{f}_q(u_q)\Big) \\
&= 2\mathrm{e}^{\beta t}\Big[u^{\mathrm{T}}(L+\bar{L})O\bar{f}(u) - u^{\mathrm{T}} LO\bar{L}u - \bar{f}^{\mathrm{T}}(u)O\bar{f}(u)\Big], \\
0 &\le 2\mathrm{e}^{\beta t} \sum_{q=1}^{m} r_q \Big(\bar{f}_q(u_{q\varrho_1}) - l_q u_{q\varrho_1}\Big)\Big(\bar{l}_q u_{q\varrho_1} - \bar{f}_q(u_{q\varrho_1})\Big) \\
&= 2\mathrm{e}^{\beta t}\Big[u_{\varrho_1}^{\mathrm{T}}(L+\bar{L})R\bar{f}(u_{\varrho_1}) - u_{\varrho_1}^{\mathrm{T}} LR\bar{L}u_{\varrho_1} - \bar{f}^{\mathrm{T}}(u_{\varrho_1})R\bar{f}(u_{\varrho_1})\Big].
\end{aligned} \tag{3.30}
$$

Based on the Newton-Leibniz formula, there exist appropriate dimensions matrices S_ϵ, U_ϵ and W_ϵ ($\epsilon = 1, 2, 3, 4$) such that

$$
\begin{aligned}
0 =& \int_G \Big\{2\Big[u^{\mathrm{T}} S_1 + u_{\varrho_1}^{\mathrm{T}} S_2 + u_{\varrho_2}^{\mathrm{T}} S_3 \\
&+ (\int_{t-\varrho_1(t)}^{t} \frac{\partial u(s,x)}{\partial s} ds)^{\mathrm{T}} S_4\Big] \cdot \Big[u - u_{\varrho_1} - \int_{t-\varrho_1(t)}^{t} \frac{\partial u(s,x)}{\partial s} ds\Big]\Big\} dx, \\
0 =& \int_G \Big\{2\Big[u^{\mathrm{T}} T_1 + u_{\varrho_1}^{\mathrm{T}} T_2 + u_{\varrho_2}^{\mathrm{T}} T_3 \\
&+ (\int_{t-\varrho_2(t)}^{t} \frac{\partial u(s,x)}{\partial s} ds)^{\mathrm{T}} T_4\Big] \cdot \Big[u - u_{\varrho_2} - \int_{t-\varrho_2(t)}^{t} \frac{\partial u(s,x)}{\partial s} ds\Big]\Big\} dx.
\end{aligned} \tag{3.31}
$$

Thus, from (3.27)-(3.31), we derive that

$$
\begin{aligned}
\frac{dV(t,u)}{dt} \le& \mathrm{e}^{\beta t} \int_G \Big\{u^{\mathrm{T}}\Big[\beta Q - 2Q\bar{\alpha} - QA - A^{\mathrm{T}}Q + P_1 + P_2 + \frac{\varrho_1 - \varrho_0}{\beta}(\mathrm{e}^{\beta\varrho_1} - \mathrm{e}^{\beta\varrho_0})P_3 \\
&- 2LO\bar{L} + 2S_1 + 2T_1\Big]u + 2u^{\mathrm{T}}\Big[QB + (L+\bar{L})O\Big]\bar{f}(u) + 2u^{\mathrm{T}}\Big[QC + (L+\bar{L})R\Big] \\
&\bar{f}(u_{\varrho_1}) + 2u^{\mathrm{T}} QD \frac{\partial u_{\varrho_2}}{\partial t} - 2u^{\mathrm{T}} S_1 u_{\varrho_1} - 2u^{\mathrm{T}} T_1 u_{\varrho_2} - 2u^{\mathrm{T}} S_1 (\int_{t-\varrho_1(t)}^{t} \frac{\partial u(s,x)}{\partial s} ds)
\end{aligned}
$$

$$
\begin{aligned}
&- 2u^{\mathrm{T}}T_1(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds) - 2\bar{f}^{\mathrm{T}}(u)O\bar{f}(u) - 2\bar{f}^{\mathrm{T}}(u_{\varrho_1})R\bar{f}(u_{\varrho_1})\\
&+ 2u_{\varrho_1}^{\mathrm{T}}(S_2+T_2)u + 2u_{\varrho_2}^{\mathrm{T}}(S_3+T_3)u - 2u_{\varrho_1}^{\mathrm{T}}T_2u_{\varrho_2} - 2u_{\varrho_2}^{\mathrm{T}}S_3u_{\varrho_1}\\
&+ u_{\varrho_1}^{\mathrm{T}}\Big[-(1-\kappa_1)\mathrm{e}^{-\beta\varrho_1}u_1 - 2S_2 - 2LR\bar{L}\Big]u_{\varrho_1} + u_{\varrho_2}^{\mathrm{T}}\Big[-(1-\kappa_2)\mathrm{e}^{-\beta\varrho_2}u_2 - 2T_3\Big]u_{\varrho_2}\\
&- 2u_{\varrho_1}^{\mathrm{T}}S_2(\int_{t-\varrho_1(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds) - 2u_{\varrho_2}^{\mathrm{T}}S_3(\int_{t-\varrho_1(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)\\
&- 2u_{\varrho_1}^{\mathrm{T}}T_2(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds) - 2u_{\varrho_2}^{\mathrm{T}}T_3(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)\\
&+ 2(\int_{t-\varrho_1(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm{T}}S_4u + 2(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm{T}}T_4u\\
&- 2(\int_{t-\varrho_1(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm{T}}S_4u_{\varrho_1} - 2(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm{T}}T_4u_{\varrho_2}\\
&- 2(\int_{t-\varrho_1(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm{T}}S_4(\int_{t-\varrho_1(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)\\
&- 2(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm{T}}T_4(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)\\
&- (\int_{t-\varrho_1}^{t-\varrho_0}u(s,x)ds)^{\mathrm{T}}P_3(\int_{t-\varrho_1}^{t-\varrho_0}u(s,x)ds) + (\frac{\partial u}{\partial t})^{\mathrm{T}}\Big[P_4 + \frac{\varrho_2}{\beta}(\mathrm{e}^{\beta\varrho_2}-1)P_5\Big](\frac{\partial u}{\partial t})\\
&- (1-\kappa_2)\mathrm{e}^{-\beta\varrho_2}(\frac{\partial u_{\varrho_2}}{\partial t})^{\mathrm{T}}P_4(\frac{\partial u_{\varrho_2}}{\partial t}) - (\int_{t-\varrho_2}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm{T}}P_5(\int_{t-\varrho_2}^{t}\frac{\partial u(s,x)}{\partial s}ds)\Big\}dx\\
=&\mathrm{e}^{\beta t}\int_G \chi^{\mathrm{T}}(t)\Xi\chi(t)dx,
\end{aligned}
\tag{3.32}
$$

where

$$
\begin{aligned}
\bar{\alpha} =&\mathrm{diag}\Big(\sum_{l=1}^{n}\frac{\alpha_{1l}}{d_l^2},\cdots,\sum_{l=1}^{n}\frac{\alpha_{ml}}{d_l^2}\Big),\\
\chi^{\mathrm{T}}(t) =&\Big[u^{\mathrm{T}}, u_{\varrho_1}^{\mathrm{T}}, u_{\varrho_2}^{\mathrm{T}}, \bar{f}^{\mathrm{T}}(u), \bar{f}^{\mathrm{T}}(u_{\varrho_1}), (\frac{\partial u}{\partial t})^{\mathrm{T}}, (\frac{\partial u_{\varrho_2}}{\partial t})^{\mathrm{T}}, (\int_{t-\varrho_1(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm{T}},\\
&(\int_{t-\varrho_2(t)}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm{T}}, (\int_{t-\varrho_1}^{t-\varrho_0}u(s,x)ds)^{\mathrm{T}}, (\int_{t-\varrho_2}^{t}\frac{\partial u(s,x)}{\partial s}ds)^{\mathrm{T}}\Big].
\end{aligned}
$$

By (3.24), we gain

$$
\frac{dV(t,u)}{dt} \le 0, \quad t \ge t_0. \tag{3.33}
$$

Thus, we have

$$
V(t,u) \le V(t_0, u(t_0,x)), \quad t \ge t_0, \tag{3.34}
$$

where

$$
\begin{aligned}
V(t_0,u(t_0,x)) =& \mathrm{e}^{\beta t_0}\int_G u^{\mathrm{T}}(t_0,x)Qu(t_0,x)dx\\
&+\int_G\int_{t_0-\varrho_1(t_0)}^{t_0}\mathrm{e}^{\beta s}u^{\mathrm{T}}(s,x)P_1u(s,x)dsdx
\end{aligned}
$$

$$
\begin{aligned}
&+\int_G\int_{t_0-\varrho_2(t_0)}^{t_0} \mathrm{e}^{\beta s}u^{\mathrm{T}}(s,x)P_2u(s,x)dsdx\\
&+(\varrho_1-\varrho_0)\int_G\int_{-\varrho_1}^{-\varrho_0}\int_{t_0+\gamma}^{t_0} \mathrm{e}^{\beta(s-\gamma)}u^{\mathrm{T}}(s,x)P_3u(s,x)dsd\gamma dx\\
&+\int_G\int_{t_0-\varrho_2(t_0)}^{t_0} \mathrm{e}^{\beta s}(\frac{\partial u(s,x)}{\partial s})^{\mathrm{T}}P_4(\frac{\partial u(s,x)}{\partial s})dsdx\\
&+\varrho_2\int_G\int_{-\varrho_2}^{0}\int_{t_0+\gamma}^{t_0} \mathrm{e}^{\beta(s-\gamma)}(\frac{\partial u(s,x)}{\partial s})^{\mathrm{T}}P_5(\frac{\partial u(s,x)}{\partial s})dsd\gamma dx
\end{aligned}
$$

$$
\begin{aligned}
\le \mathrm{e}^{\beta t_0}\Big\{&\lambda_{\max}(Q)+\lambda_{\max}(P_1)\frac{1-\mathrm{e}^{-\beta\varrho_1}}{\beta}+\lambda_{\max}(P_2)\frac{1-\mathrm{e}^{-\beta\varrho_2}}{\beta}\\
&+\lambda_{\max}(P_3)\Big[\frac{\varrho_1-\varrho_0}{\beta^2}(\mathrm{e}^{\beta\varrho_1}-\mathrm{e}^{\beta\varrho_0})]-\frac{(\varrho_1-\varrho_0)^2}{\beta}\Big]+\lambda_{\max}(P_4)\frac{1-\mathrm{e}^{-\beta\varrho_2}}{\beta}\\
&+\lambda_{\max}(P_5)\Big[\frac{\varrho_2}{\beta^2}(\mathrm{e}^{\beta\varrho_2}-1)-\frac{\varrho_2^2}{\beta}\Big]\Big\}\sup_{-\varrho\le\vartheta\le0}\|\varpi(\vartheta,x)\|^2.
\end{aligned}
\tag{3.35}
$$

Noticing that

$$
V(t,u)\ge\lambda_{\min}(Q)\mathrm{e}^{\beta t}\|u\|^2,\quad t\ge t_0. \tag{3.36}
$$

Therefore, derive from (3.34)-(3.36)

$$
\|u\|^2 \le \mathrm{e}^{-\beta(t-t_0)}\rho\sup_{-\varrho\le\vartheta\le0}\|\varpi(\vartheta,x)\|^2, \tag{3.37}
$$

where

$$
\begin{aligned}
\rho=&\frac{\bar\rho}{\lambda_{\min}(Q)},\\
\bar\rho=&\lambda_{\max}(Q)+\lambda_{\max}(P_1)\frac{1-\mathrm{e}^{-\beta\varrho_1}}{\beta}+\lambda_{\max}(P_2)\frac{1-\mathrm{e}^{-\beta\varrho_2}}{\beta}\\
&+\lambda_{\max}(P_3)\Big[\frac{\varrho_1-\varrho_0}{\beta^2}(\mathrm{e}^{\beta\varrho_1}-\mathrm{e}^{\beta\varrho_0})-\frac{(\varrho_1-\varrho_0)^2}{\beta}\Big]\\
&+\lambda_{\max}(P_4)\frac{1-\mathrm{e}^{-\beta\varrho_2}}{\beta}+\lambda_{\max}(P_5)\Big[\frac{\varrho_2}{\beta^2}(\mathrm{e}^{\beta\varrho_2}-1)-\frac{\varrho_2^2}{\beta}\Big].
\end{aligned}
$$

Thus, $u(t,x)=0$ of (3.23) is exponentially stable. Proof is end. □

Remark 3.1. Stability analysis of SDEs and NSDEs without impulse effects, Markovian switching and time-varying delays are shown in [23]. While in [6], the exponential stability for time-varying delayed NSDEs without reaction-diffusion terms are obtained. The authors in [3,19,26] did not consider impulsive effects and reaction-diffusion terms had been obtained. In [7,21], the stability criteria for NSDEs with impulse effects are considered. By proposing the asymptotic stability analysis of delayed neutral-type NNs without Markovian switching and stochastic perturbation are given in [24,28]. In [15,16], the impulsive systems without neutral-type, stochastic perturbation and Markovian switching have been considered. [8] did not considered reaction-diffusion terms. We considered the stability analysis of Markovian switching time-varying delayed reaction-diffusion neutral-type stochastic NNs. Thus, the previous models and results are special cases of ours. Our findings are new.

4. Examples

Example 4.1. To verify the conclusion, we consider a 2-dimensional Markovian switching neutral-type reaction-diffusion stochastic differential equations. For system (2.6) , Markov chain $r(t)$, $t \geq 0$ is taking values in a finite state space $S = \{1, 2\}$ with generator

$$\Pi = (\pi_{ij})_{2\times 2} = \begin{bmatrix} -0.56 & 0.56 \\ 0.34 & -0.34 \end{bmatrix}. \tag{4.1}$$

We assume that

$$\beta = 0.4, \quad \alpha_l(t, x, u) = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad l = 1,$$

$$A_1 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.8 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 0.4 \end{bmatrix}, \; C_1 = \begin{bmatrix} 0.4 & 0.2 \\ 0.3 & 0.4 \end{bmatrix}, \; D_1 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1.4 & 0 \\ 0 & 1.8 \end{bmatrix}, \; B_2 = \begin{bmatrix} -2.2 & 1.1 \\ -1.2 & 1.4 \end{bmatrix}, \; C_2 = \begin{bmatrix} 2.4 & -3.2 \\ 2.3 & -2.4 \end{bmatrix}, \; D_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix},$$

$$\bar{f}(u) = 0.2\tanh(u), \; \varrho_1(t) = 0.8 + 0.2\sin(4t), \; \varrho_2(t) = 0.7 + 0.3\cos(6t),$$

$$g_1 = g_2 = \begin{bmatrix} 0.3u_1 & 0 \\ 0 & 0.2u_{2\varrho_1} \end{bmatrix} + \begin{bmatrix} 0.3u_2 & 0 \\ 0 & 0.2u_{2\varrho_2} \end{bmatrix},$$

Then, from above parameters, we derive that system (2.6) satisfies hypotheses (H_1) and (H_2) with

$$\varrho_0 = 0, \; \varrho_1 = 1, \; \varrho_2 = 0.8, \; \kappa_1 = 1.2, \; \kappa_2 = 1.8, \; \varrho = 1, \; L = 0, \; \bar{L} = 0.2I,$$
$$M_{11} = 0.18I, \; M_{21} = 0.18I, \; M_{31} = 0.18I, \; M_{12} = 0.08I, \; M_{22} = 0.08I, \; M_{32} = 0.08I.$$

If $r(t) = i = 1$, by (3.1)-(3.2) and Matlab LMI toolbox, we obtain that $\lambda_1 = 2.9555e + 01$.

$$Q_1 = \begin{bmatrix} -3.9665e+00 & 1.0040e-01 \\ 1.0040e-01 & -4.5730e+00 \end{bmatrix}, \; Q_2 = \begin{bmatrix} 5.9636e+02 & 3.1555e-01 \\ 3.1555e-01 & 5.8835e+02 \end{bmatrix},$$

$$O_1 = \begin{bmatrix} 3.6088e+01 & 1.4960e-01 \\ 1.4960e-01 & 3.6339e+01 \end{bmatrix}, \; R_1 = \begin{bmatrix} 1.6581e+01 & 0 \\ 0 & 1.6581e+01 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} -3.5506e+02 & 1.2670e-13 \\ 1.2670e-13 & -3.5506e+02 \end{bmatrix}, \; P_2 = \begin{bmatrix} -8.1940e+01 & 6.4146e-15 \\ 6.4146e-15 & -8.1940e+01 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 3.2240e+01 & 1.4646e-01 \\ 1.4646e-01 & 3.2485e+01 \end{bmatrix},\ P_4 = \begin{bmatrix} 3.3825e+01 & 3.6637e-15 \\ 3.6637e-15 & 3.3825e+01 \end{bmatrix},$$

$$P_5 = \begin{bmatrix} -5.8953e+01 & 0 \\ 0 & -5.8953e+01 \end{bmatrix},\ P_6 = \begin{bmatrix} 3.3640e+01 & 0 \\ 0 & 3.3640e+01 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 4.2281e+00 & 2.0517e-15 \\ 2.0517e-15 & 4.2281e+00 \end{bmatrix},\ S_2 = \begin{bmatrix} -4.2281e+00 & 5.2649e-15 \\ 5.2649e-15 & -4.2281e+00 \end{bmatrix},$$

$$S_3 = \begin{bmatrix} 2.1140e+00 & 4.4059e-16 \\ 4.4059e-16 & 2.1140e+00 \end{bmatrix},\ S_4 = \begin{bmatrix} 1.0570e+01 & -8.0328e-16 \\ -8.0328e-16 & 1.0570e+01 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 4.2281e+00 & 1.2251e-15 \\ 1.2251e-15 & 4.2281e+00 \end{bmatrix},\ T_2 = \begin{bmatrix} 2.1140e+00 & -1.2179e-15 \\ -1.2179e-15 & 2.1140e+00 \end{bmatrix},$$

$$T_3 = \begin{bmatrix} -4.2281e+00 & 1.1212e-15 \\ 1.1212e-15 & -4.2281e+00 \end{bmatrix},\ T_4 = \begin{bmatrix} 1.0570e+01 & 2.5968e-17 \\ 2.5968e-17 & 1.0570e+01 \end{bmatrix}.$$

If $r(t) = i = 2$, by (3.1)-(3.2) and Matlab LMI toolbox, we obtain that $\lambda_2 = 3.3783e+01$.

$$Q_1 = \begin{bmatrix} 9.4092e+02 & 4.4733e-01 \\ 4.4733e-01 & 9.2635e+02 \end{bmatrix},\ Q_2 = \begin{bmatrix} 6.7074e-01 & 4.2964e-02 \\ 4.2964e-02 & -7.2254e-01 \end{bmatrix},$$

$$O_2 = \begin{bmatrix} 3.6252e+01 & -1.8780e-01 \\ -1.8780e-01 & 3.6117e+01 \end{bmatrix},\ R_2 = \begin{bmatrix} 1.6573e+01 & 0 \\ 0 & 1.6573e+01 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} -3.3539e+02 & 1.1173e-14 \\ 1.1173e-14 & -3.3539e+02 \end{bmatrix},\ P_2 = \begin{bmatrix} -7.7401e+01 & 5.6825e-16 \\ 5.6825e-16 & -7.7401e+01 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 3.2401e+01 & -1.8386e-01 \\ -1.8386e-01 & 3.2270e+01 \end{bmatrix},\ P_4 = \begin{bmatrix} 3.3809e+01 & 7.6319e-17 \\ 7.6319e-17 & 3.3809e+01 \end{bmatrix},$$

$$P_5 = \begin{bmatrix} -5.8925e+01 & 0 \\ 0 & -5.8925e+01 \end{bmatrix},\ P_6 = \begin{bmatrix} 3.3624e+01 & 0 \\ 0 & 3.3624e+01 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 4.2261e+00 & 1.8131e-16 \\ 1.8131e-16 & 4.2261e+00 \end{bmatrix},\ S_2 = \begin{bmatrix} -4.2261e+00 & 4.6428e-16 \\ 4.6428e-16 & -4.2261e+00 \end{bmatrix},$$

$$S_3 = \begin{bmatrix} 2.1131e+00 & 3.8732e-17 \\ 3.8732e-17 & 2.1131e+00 \end{bmatrix},\ S_4 = \begin{bmatrix} 1.0565e+01 & -7.0742e-17 \\ -7.0742e-17 & 1.0565e+01 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 4.2261e+00 & 1.0833e-16 \\ 1.0833e-16 & 4.2261e+00 \end{bmatrix},\ T_2 = \begin{bmatrix} 2.1131e+00 & -1.0723e-16 \\ -1.0723e-16 & 2.1131e+00 \end{bmatrix},$$

$$T_3 = \begin{bmatrix} -4.2261e+00 & 9.9367e-17 \\ 9.9367e-17 & -4.2261e+00 \end{bmatrix},\ T_4 = \begin{bmatrix} 1.0565e+01 & 2.2407e-18 \\ 2.2407e-18 & 1.0565e+01 \end{bmatrix}.$$

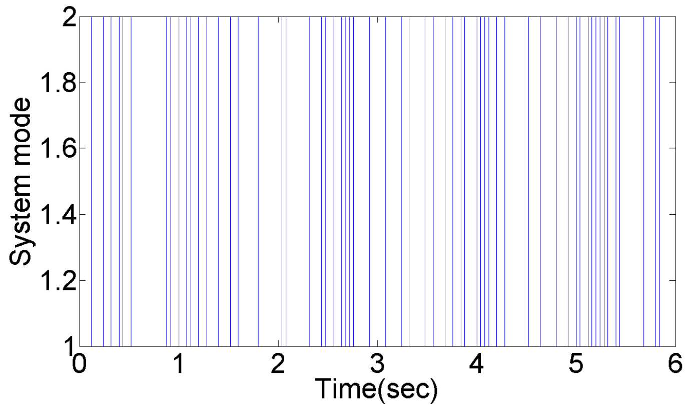

Figure 1. System switching mode

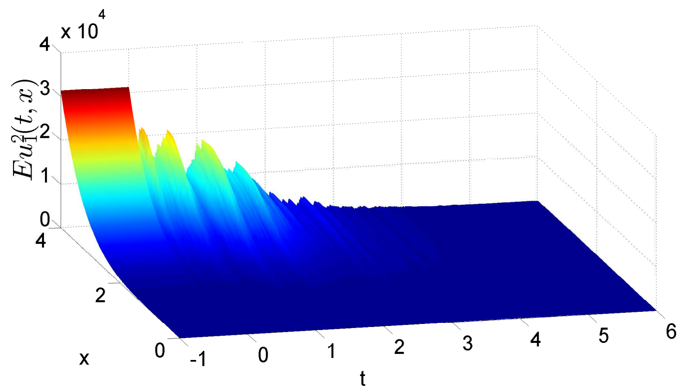

Figure 2. State $u_1(t,x)$ mean square simulation of system (8) with $r(t)=1$

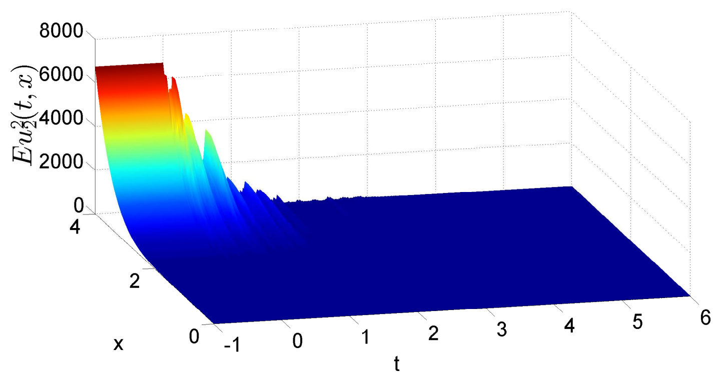

Figure 3. State $u_2(t,x)$ mean square simulation of system (8) with $r(t)=1$

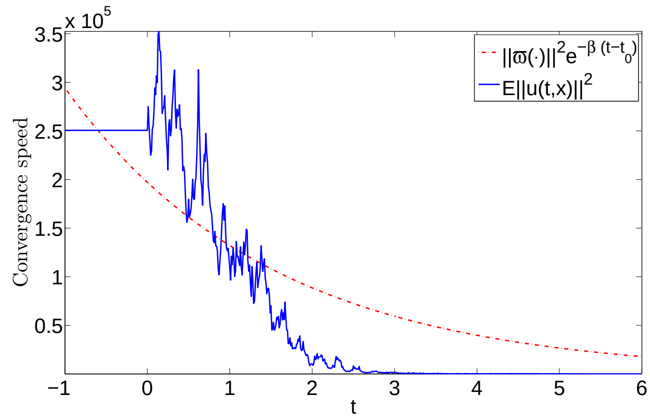

Figure 4. Convergence of $\|\varpi(\cdot)\|^2 e^{-\beta(t-t_0)}$ and $E\|u(t,x)\|^2$ with $r(t) = 1$

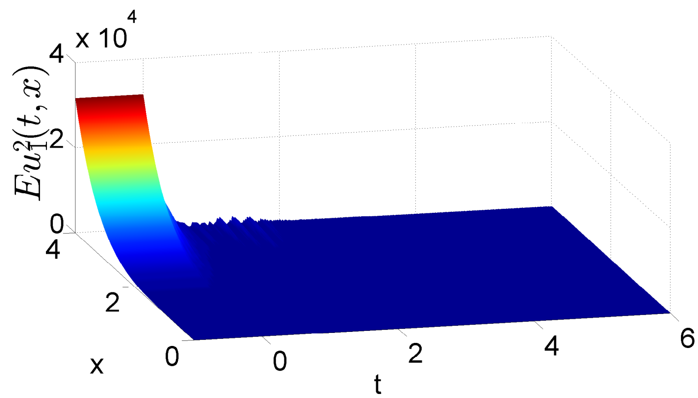

Figure 5. State $u_1(t,x)$ mean square simulation of system (8) with $r(t) = 2$

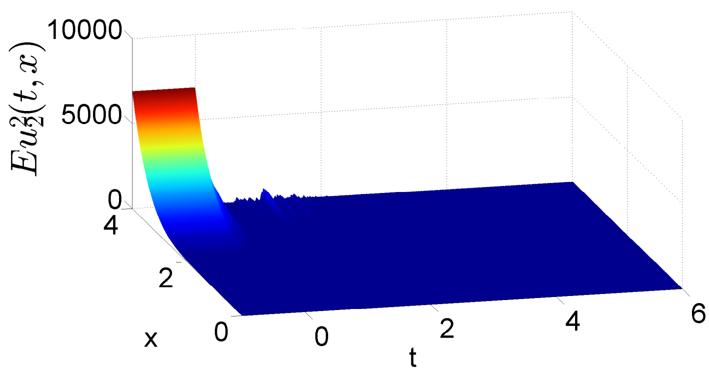

Figure 6. State $u_2(t,x)$ mean square simulation of system (8) with $r(t) = 2$

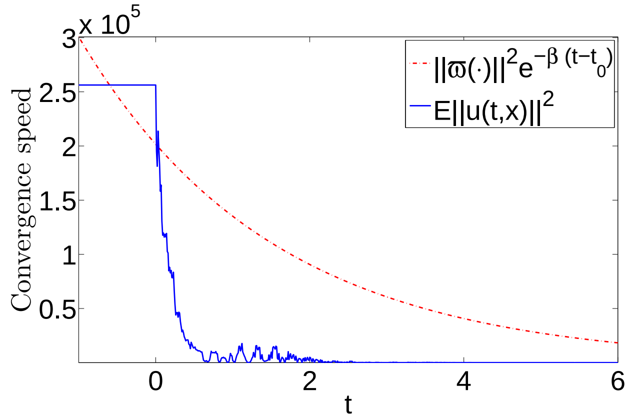

Figure 7. Convergence of $\|\varpi(\cdot)\|^2 e^{-\beta(t-t_0)}$ and $E\|u(t,x)\|^2$ with $r(t)=2$

Figure 1 indicates the system switching mode. Figure 2 and Figure 3 show the simulations of state $Eu_1^2(t,x)$ and $Eu_2^2(t,x)$ with $r(t)=1$ respectively. Figure 4 diplays the convergence of $\|\varpi(\cdot)\|^2 e^{-\beta(t-t_0)}$ and $E\|u(t,x)\|^2$ with $r(t)=1$. Figure 5 and Figure 6 show the simulations of state $Eu_1^2(t,x)$ and $Eu_2^2(t,x)$ with $r(t)=2$ respectively. Figure 7 displays the convergence of $\|\varpi(\cdot)\|^2 e^{-\beta(t-t_0)}$ and $E\|u(t,x)\|^2$ with $r(t)=2$. Thus, by Theorem 3.1, we derive that $u(t,x)=0$ of (2.6) is exponential stable in mean square.

Example 4.2. For system (3.23), we suppose that

$$A=\begin{bmatrix}0.2 & 0\\ 0 & 0.8\end{bmatrix},\ B=\begin{bmatrix}0.18 & 0.2\\ 0.2 & 0.3\end{bmatrix},\ C=\begin{bmatrix}0.6 & 0.1\\ 0.3 & 0.4\end{bmatrix},\ D=\begin{bmatrix}0.4 & 0\\ 0 & 0.4\end{bmatrix},$$
$$\bar{f}(u)=0.2\tanh(u),$$
$$\varrho_1(t)=0.6+0.4\sin(3t),\ \ \varrho_2(t)=0.6+0.2\cos(6t),$$

Then, from above parameters, we derive that system (2.6) satisfies hypotheses (H_1) and (H_2) with

$$\varrho_0=0,\ \varrho_1=1,\ \varrho_2=0.8,\ \kappa_1=1.2,\ \kappa_2=1.2,\ \varrho=1,\ L=0,\ \bar{L}=0.2I.$$

Let $\beta=0.04$. By the Matlab LMI toolbox and (3.24) in Theorem 2, we obtain that

$$Q=\begin{bmatrix}-1.7739e-01 & -8.2555e-02\\ -8.2555e-02 & -4.6801e-01\end{bmatrix},\ O=\begin{bmatrix}3.3889e+00 & 1.8724e-01\\ 1.8724e-01 & 3.6258e+00\end{bmatrix},$$
$$R=\begin{bmatrix}7.5375e-01 & 0\\ 0 & 7.5375e-01\end{bmatrix},\ P_1=\begin{bmatrix}-1.7572e+00 & -6.6990e-02\\ -6.6990e-02 & -2.1097e+00\end{bmatrix},$$
$$P_2=\begin{bmatrix}-2.6038e+00 & -5.6448e-02\\ -5.6448e-02 & -2.9009e+00\end{bmatrix},\ P_3=\begin{bmatrix}8.5603e-01 & 5.8244e-02\\ 5.8244e-02 & 9.2967e-01\end{bmatrix},$$
$$P_4=\begin{bmatrix}-2.9588e+00 & 0\\ 0 & -2.9588e+00\end{bmatrix},\ P_5=\begin{bmatrix}1.6588e+00 & 0\\ 0 & 1.6588e+00\end{bmatrix},$$

$$S_1 = \begin{bmatrix} 4.1915e-01 & -1.1813e-03 \\ -1.1813e-03 & 4.1293e-01 \end{bmatrix}, \; S_2 = \begin{bmatrix} 3.2466e-01 & -2.6905e-03 \\ -2.6905e-03 & 3.1050e-01 \end{bmatrix},$$

$$S_3 = \begin{bmatrix} 3.6863e-02 & 3.0837e-04 \\ 3.0837e-04 & 3.8486e-02 \end{bmatrix}, \; S_4 = \begin{bmatrix} 4.0803e-01 & 3.7730e-04 \\ 3.7730e-04 & 4.1002e-01 \end{bmatrix},$$

$$T_1 = \begin{bmatrix} 4.1032e-01 & -1.1354e-03 \\ -1.1354e-03 & 4.0434e-01 \end{bmatrix}, \; T_2 = \begin{bmatrix} 1.9207e-02 & 4.0027e-04 \\ 4.0027e-04 & 2.1314e-02 \end{bmatrix},$$

$$T_3 = \begin{bmatrix} 2.8052e-01 & -2.4607e-03 \\ -2.4607e-03 & 2.6757e-01 \end{bmatrix}, \; T_4 = \begin{bmatrix} 4.1686e-01 & 3.3134e-04 \\ 3.3134e-04 & 4.1860e-01 \end{bmatrix}.$$

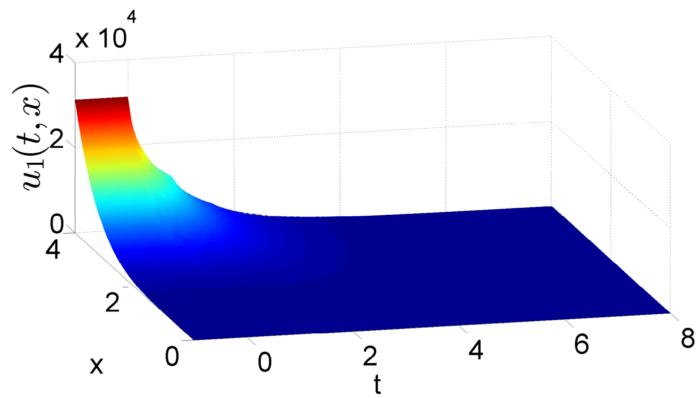

Figure 8. State $u_1(t,x)$ simulation of system (38)

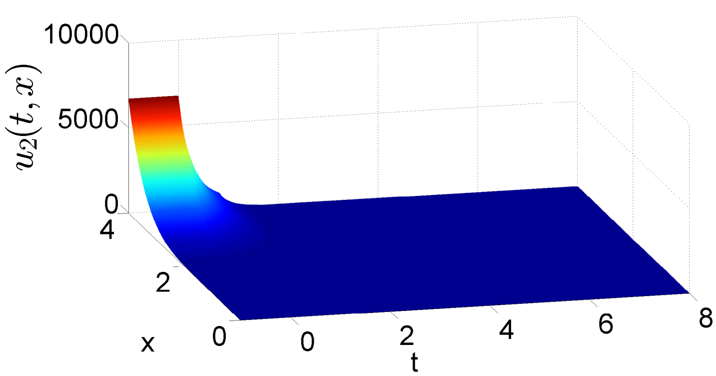

Figure 9. State $u_2(t,x)$ simulation of system (38)

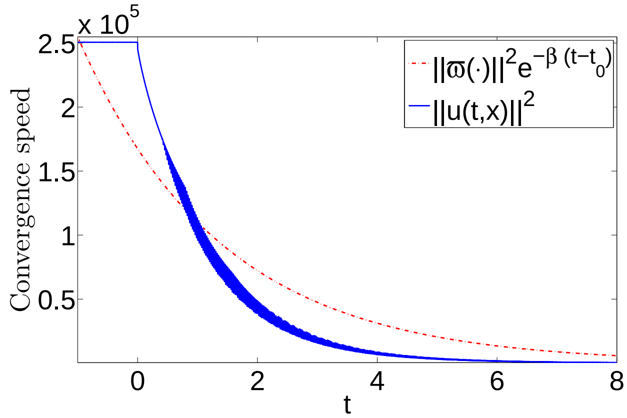

Figure 10. Convergence of $\|\varpi(\cdot)\|^2 e^{-\beta(t-t_0)}$ and $\|u(t,x)\|^2$

Figure 8 and Figure 9 show the trajectories of state $u_1(t,x)$ and $u_2(t,x)$ respectively. Figure 10 displays the convergence of $\|\varpi(\cdot)\|^2 e^{-\beta(t-t_0)}$ and $\|u(t,x)\|^2$. Therefore, by Theorem 3.2, we derive that $u(t,x) = 0$ of (3.24) is exponentially stable.

5. Conclusions

In this paper, we have considered the exponential mean square stability problem of Markovian switching delayed reaction-diffusion neutral-type stochastic neural networks via LMI technique. By constructing the Lyapunov functional, several new criteria on the exponential mean square stability of delayed Markovian switching RNSNNs are provided. The synchronization of Markovian switching RNSNNs with impulses and the high order impulsive fractional differential problem will be studied in our future work [14, 20, 33].

References

[1] P. Balasubramaniam and C. Vidhya, *Exponential stability of stochastic reaction–diffusion uncertain fuzzy neural networks with mixed delays and Markovian jumping parameters*, Expert Systems with Applications, 2012, 39, 3109–3115.

[2] Y. Cao, Y. Kao, J. Park and H. Bao, *Global Mittag-Leffler stability of the delayed fractional-coupled reaction–diffusion system on networks without strong connectedness*, IEEE Transactions on Neural Networks and Learning Systems, 2021, 11 pages. DOI: 10.1109/TNNLS.2021.3080830

[3] Y. Cao, R. Samidurai and R. Sriraman, *Stability and dissipativity analysis for neutral type stochastic Markovian jump static neural networks with time delays*, Journal of Artificial Intelligence and Soft Computing Research, 2019, 9(3), 189–204.

[4] P. Cheng and F. Deng, *Global exponential stability of impulsive stochastic functional differential systems*, Statistics and Probability Letters, 2020, 80, 1854–1862.

[5] R. Douaifia, S. Abdelmalek and S. Bendoukha, *Global existence and asymptotic stability for a class of coupled reaction–diffusion systems on growing domains*, Acta Applicandae Mathematicae, 2021, 171, 17, 13 pages.

[6] L. Feng, Z. Wu, J. Cao, S. Zheng and F. E. Alsaadi, *Exponential stability for nonlinear hybrid stochastic systems with time varying delays of neutral type*, Applied Mathematics Letters, 2020, 107, Article ID 106468, 9 pages.

[7] X. Fu, Q. Zhu, *Exponential stability of neutral stochastic delay differential equation with delay-dependent impulses*, Applied Mathematics and Computation, 2020, 377, Article ID 125146, 9 pages.

[8] Y. Gao, W. Zhou, C. Ji, D. Tong and J. Fang, *Globally exponential stability of stochastic neutral-type delayed neural networks with impulsive perturbations and Markovian switching*, Nonlinear dynamics, 2012, 70(3), 2107–2116.

[9] Y. Kao, J. Guo, C. Wang and X. Sun, *Delay-dependent robust exponential stability of Markovian jumping reaction–diffusion Cohen-Grossberg neural networks with mixed-delays*, Journal of the Franklin Institute, 2012, 349, 1972–1988.

[10] Y. Kao, C. Wang, H. R. Karimi and R. Bi, *Global stability of coupled Markovian switching reaction–diffusion systems on networks*, Nonlinear Analysis: Hybrid Systems, 2014, 13, 61–73.

[11] Y. Kao, C. Wang, F. Zha and H. Cao, *Stability in mean of partial variables for stochastic reaction–diffusion systems with Markovian switching*, Journal of the Franklin Institute, 2014, 35, 500–512.

[12] Y. Kao, C. Wang and L. Zhang, *Delay-dependent robust exponential stability of impulsive Markovian jumping reaction–diffusion Cohen–Grossberg neural networks*, Neural Processing Letters, 2013, 38, 321–346.

[13] Y. Kao, Q. Zhu and W. Qi, *Exponential stability and instability of impulsive stochastic functional differential equations with Markovian switching*, Applied Mathematics and Computation, 2015, 271, 795–804.

[14] F. Jiang, *Existence of Periodic Solutions in Impulsive Differential Equations*, Journal of Nonlinear Modeling and Analysis, 2021, 3(1), 53–70.

[15] X. Li, D. W. C. Ho and J. Cao, *Finite-time stability and settling-time estimation of nonlinear impulsive systems*, Automatica, 2019, 99, 361–368.

[16] X. Li, X. Yang and T. Huang, *Persistence of delayed cooperative models: Impulsive control method*, Applied Mathematics and Computation, 2019, 342, 130–146.

[17] Y. Li, W. Fei and S. Deng, *Delay feedback stabilization of stochastic differential equations driven by G-Brownian motion*, International Journal of Control, 2021. DOI: 10.1080/00207179.2021.1916077

[18] Y. Li and Y. Ren, *Stability for stochastic reaction–diffusion systems driven by G-Brownian motion*, International Journal of Control, 2021. DOI: 10.1080/00207179.2021.1885742

[19] Z. Li, S. Shang and J. Lam, *On stability of neutral-type linear stochastic time-delay systems with three different delays*, Applied Mathematics and Computation, 2019, 360, 147–166.

[20] Z. Li and T. Zhang, *Analysis of a Kind of Stochastic Dynamics Model with Nonlinear Function*, Journal of Nonlinear Modeling and Analysis, 2021, 3(4), 523–546.

[21] Y. Liu and D. Ruan, *Stability of a class of impulsive neutral stochastic functional partial differential equations*, Discrete Dynamics in Nature and Society, 2020(2), 1–12.

[22] Y. Lv, W. Lv and J. Sun, *Convergence dynamics of stochastic reaction-diffusion recurrent neural networks with continuously distributed delays*, Nonlinear Analysis: Real World Applications, 2008, 9, 1590–1606.

[23] X. Mao, *Stochastic differential equations and applications*, Horwood Publishing Limited, Chichester, 2007.

[24] N. Ozcan, *Stability analysis of Cohen–Grossberg neural networks of neutral-type: Multiple delays case*, Neural Networks, 2019, 113, 20–27.

[25] Y. Ren, W. Yin and R. Sakthivel, *Stabilization of stochastic differential equations driven by G-Brownian motion with feedback control based on discrete-time state observation*, Automatica, 2018, 95, 146–151.

[26] M. Shen, W. Fei, X. Mao and S. Deng, *Exponential stability of highly nonlinear neutral pantograph stochastic differential equations*, Asian Journal of Control, 2018, 22(4), 1–12.

[27] Y. Shi and P. Zhu, *Asymptotic stability analysis of stochastic reaction–diffusion Cohen–Grossberg neural networks with mixed time delays*, Applied Mathematics and Computation, 2014, 242, 159–167.

[28] R. Sriraman and R. Samidurai, *Global asymptotic stability analysis for neutral-type complex-valued neural networks with random time-varying delays*, International Journal of Systems Science, 2019, 50(9), 1742–1756.

[29] K. Sun, H. Yu and X. Xia, *Distributed control of nonlinear stochastic multi-agent systems with external disturbance and time-delay via event-triggered strategy*, Neurocomputing, 2021, 452, 275–283.

[30] Suriguga, Y. Kao, C. Shao and X. Chen, *Stability of high-order delayed Markovian jumping reaction–diffusion HNNs with uncertain transition rates*, Applied Mathematics and Computation, 2021, 389, Article ID 125559, 16 pages.

[31] L. Wan and Q. Zhou, *Exponential stability of stochastic reaction-diffusion Cohen–Grossberg neural networks with delays*, Applied Mathematics and Computation, 2008, 206, 818–824.

[32] C. Wang, Y. Kao and G. Yang, *Exponential stability of impulsive stochastic fuzzy reaction–diffusion Cohen–Grossberg neural networks with mixed delays*, Neurocomputing, 2012, 89, 55–63.

[33] D. Wang and J. Jiang, *Existence and Multiplicity of Positive Solutions for a Singular Nonlinear High Order Fractional Differential Problem with Multipoint Boundary Conditions*, Journal of Nonlinear Modeling and Analysis, 2021, 3(4), 603–616.

[34] L. Wang, Z. Zhang and Y. Wang, *Stochastic exponential stability of the delayed reaction-diffusion recurrent neural networks with Markovian jumping parameters*, Physics Letters A, 2008, 372, 3201–3209.

[35] P. Wang, X. Wang and H. Su, *Input-to-state stability of impulsive stochastic infinite dimensional systems with Poisson jumps*, Automatica, 2021, 128(2107), Article ID 109553, 9 pages.

[36] T. Wei, X. Li and V. Stojanovic, *Input-to-state stability of impulsive reaction-diffusion neural networks with infinite distributed delays*, Nonlinear Dynamics, 2021, 103, 1733–1755.

[37] T. Wu, L. Xiong, J. Cao, J. Park and J. Cheng, *Synchronization of coupled reaction–diffusion stochastic neural networks with time-varying delay via delay-dependent impulsive pinning control algorithm*, Communications in Nonlinear Science and Numerical Simulation, 2021, 99, Article ID 105777, 21 pages.

[38] X. Wu, S. Liu, Y. Wang and Z. Liu, *Asymptotical stability of fractional neutral-type delayed neural networks with reaction–diffusion terms*, Neurocomputing, 2021, 461(1), 77–85.

[39] C. Xiao and T. Hou, *Stabilization and event-triggered control of stochastic delay systems with Markovian jump parameters*, IEEE Access, 2020, 8, 193569–193577.

[40] J. Xie, Y. Kao, C. Wang and C. Gao, *Delay-dependent robust stability of uncertain neutral-type Itô stochastic systems with Markovian jumping parameters*, Applied Mathematics and Computation, 2015, 251, 576–585.