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A Mathematical Study for the Stability of Two Predator and One Prey with Infection in First Predator using Fuzzy Impulsive Control

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Abstract. In this study, we develop a set of ordinary differential equations that represents the dynamics of an ecosystem with two predators and one prey, but only the first predator population is affected by an infectious disease. The Lotka-Volterra predator-prey system's model stability have been examined using the Takagi-Sugeno (T-S) impulsive control model and the Fuzzy impulsive control model. Following the formulation of the model, the global stabilities and the Fuzzy solution are carried out through numerical simulations and graphical representations with appropriate discussion for better understanding the dynamics of our proposed model.

AMS subject classifications: 92D25, 92D30, 92D40, 49N25

Key words: Prey-predator system, T-S model, stability, eco-epidemiology.

1 Introduction

One of the most popular subjects in biomathematics is population dynamics. There has always been an unique interest in the study of population evolution, beginning with populations of a single species and progressing to more realistic models

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where various species coexist and communicate with one another in the same ecosystem. Between these, we can find models that look at predator-prey relationships, symbiosis, or competitive connections. Since the well-known Lotka-Volterra model was developed and the major issues with ecological processes were resolved [1, 36], mathematical models are frequently used by applied mathematicians to analyse the intricate interactions between predators and prey. The classical ecological models of interacting populations typically have focused on two species. The literature has looked into continuous time models of two interacting species in great detail [16]. These models can only display the following two basic patterns mathematically: approach to a limit cycle or a steady state [17]. However, it has been found that ecological groups in nature have extremely complicated dynamical tendencies. According to Price et al. [27] community behaviour needs to be based on at least three trophic levels. There are reports of more intricate patterns in three species continuous time models [2, 14, 20, 21, 26, 32, 33, 38].

One of the most intriguing areas in mathematical biology is the interaction between predators and prey. The well-known Lotka-Volterra predator-prey model is the first mathematical representation of the interaction between predators and prey [37], which is a two-species model. Some scholars have noted that population models with two species can't accurately capture the real world [15, 31], and models with three or more species can only depict a significant number of crucial behaviours. The advancement of mathematics also demonstrated that three-species food chain models have significantly more detailed features than two-species models [8, 34].

Since the impact of infectious diseases on the ecological system regulates population size, researchers have recently become more interested in the fusion of ecology and epidemiology. There are a lot of prey-predator models that have infectious infections. The dynamics of the prey-predator system with disease in the prey and predator populations were hypothesised and examined by [6, 7], Haque and Venturino [22], Haque et al. [23, 24], Xiao and Chen [40, 41], Zhou et al. [42], Tewa [12], Hethcote [9], Hudson [28], recently, Deng [25] etc. Additionally, numerous research studies have explored the dynamic behaviour of the predator-prey system with infection in the predator population. [7]. There are also several scholars who have studied eco-epidemic models where predator populations are infected through consuming prey, such as Anderson and May [30], Hadeler and Freedman [13] etc. The dynamics of a predator-prey model with disease in both prey and predator populations were proposed by Hsieh and Hsiao [43]. Additionally, some researchers have developed eco-epidemic models with optimal control [3] and with temporal delays [11, 18].

We have witnessed rapidly growing interested in fuzzy control in recent years. This is largely sparked by the numerous successful applications fuzzy control has enjoyed. Despite the visible success, it has been made aware that many basic issues remain to be addressed. Among them, stability analysis, systematic design, and performance analysis, to name a few, are crucial to the validity and applicability of any control design methodology [5, 35]. However, it should be admitted that the stability of fuzzy logic controller (FLC) is still an open problem. It is important to point out that there exist many systems, like the predator-prey system, which cannot commonly endure continuous control inputs, or they have impulsive dynamical behavior due to abrupt jumps at certain instants during the evolving processes. Hence, it is necessary to extend FLC and reflect these impulsive jump phenomena in the predator-prey system. As on date a very few papers discussed about the stability of two dimensional Lotka-Volterra predator-prey system with fuzzy impulsive control [45].

In this paper, we have considered Lotka-Volterra predator-prey model with one prey and two predator. We also consider that only first predator population is infected by an infectious disease, i.e., the first predator population is divided into two sub-classes: susceptible and infected. To improve the model's reality we analyze the global and asymptotic stability as given in [4,29,44] of this model with the help of the T-S model, then presented the graphical solutions for the problem by considerations. Only a few articles have looked at the stability of the Lotka-Volterra predator-prey system with fuzzy impulsive control so far. So, using the T-S mathematical model and fuzzy impulsive control, the stability of the predator-prey system is examined with the help of [19, 39, 45].

2 Model formation

Our mathematical model is based on the following assumptions:

- Let x be the total population density of the prey.
- The initial group of predators is the only one to have an infectious disease.
- The overall population of first predators is split into two subclasses when a disease is present: (i) the susceptible first predator population (y_s) and (ii) the infected first predator population (y_i).
- According to the rule of mass action, the disease in the first predator population is spread horizontally from the susceptible to the infected first predator population at a constant rate of infection β.
- Let the second predator total population density is denoted by z.

- Let t be the number of years.

The following model is proposed utilising a set of non-linear ordinary differential equations based on the aforementioned presumptions.

$$\frac{dx}{dt}=rx-ex^2-\frac{P_1y_sx}{a_0+x}-\frac{P_2y_ix}{a_0+x}-\frac{P_3zx}{a_0+x}, \tag{2.1a}$$

$$\frac{dy_s}{dt}=\frac{C_1P_1y_sx}{a_0+x}-\frac{C_1P_1y_sz}{a_1+x}-\beta y_sy_i-m_1y_s, \tag{2.1b}$$

$$\frac{dy_i}{dt}=\beta y_sy_i+\frac{C_2P_2y_ix}{a_0+x}-\frac{C_2P_2y_iz}{a_2+x}-m_2y_i, \tag{2.1c}$$

$$\frac{dz}{dt}=\frac{C_3P_3xz}{a_0+x}-\frac{C_3P_3y_sz}{a_1+x}+\frac{C_3P_3y_iz}{a_2+x}-m_3z, \tag{2.1d}$$

where all of the parameters are positive and initial conditions are $x(0)>0$, $y_s(0)>0$, $y_i(0)>0$, $z(0)>0$. Here r is intrinsic growth rate of prey, e is intra-specific competition, β is infection transmission, P_1 is predation rate of susceptible first predator, P_2 is predation rate of infected first predator, P_3 is predation rate of second predator, C_1 is efficiency of first susceptible predator, C_2 is conversion efficiency of first infected predator, C_3 is conversion efficiency of second predator, m_1 mortality rate of first susceptible predator, m_2 mortality rate of first infected predator, m_3 mortality rate of second predator, a_0, a_1, a_2 are half-saturation constants, $\mathcal{R}_0=\frac{\beta}{m_2}$ is the basic reproduction number.

A matrix differential equation is stated as follows to analyse the system's stability:

$$\dot{x}=Ax+\phi(x), \tag{2.2}$$

where

$$\dot{x}=\begin{pmatrix}\dot{x}(t)\\ \dot{y_s}(t)\\ \dot{y_i}(t)\\ \dot{z}(t)\end{pmatrix},\quad A=\begin{bmatrix}r&0&0&0\\0&-m_1&0&0\\0&0&-m_2&0\\0&0&0&-m_3\end{bmatrix},$$

$$\phi(x)=\begin{bmatrix}-ex^2-\dfrac{P_1y_sx}{a_0+x}-\dfrac{P_2y_ix}{a_0+x}-\dfrac{P_3zx}{a_0+x}\\ \dfrac{C_1P_1y_sx}{a_0+x}-\dfrac{C_1P_1y_sz}{a_1+x}-\beta y_sy_i\\ \beta y_sy_i+\dfrac{C_2P_2y_ix}{a_0+x}-\dfrac{C_2P_2y_iz}{a_2+x}\\ \dfrac{C_3P_3zx}{a_0+x}-\dfrac{C_3P_3y_sz}{a_1+x}+\dfrac{C_3P_3y_iz}{a_2+x}\end{bmatrix}.$$

3 T-S fuzzy model with impulsive effects

3.1 Lemma

$\dot{x}=f(x(t))$, here the state variable is $x(t)\in R^n$, and $f\in C[R^n,R^n]$ fulfils the condition $f(0)=0$, is a compact vector field defined in $W\subseteq Rn$. Using the techniques proposed by Tanaka and Wang [10], We can build a fuzzy model for system 2.1 as shown below.

Control Rule i $(i=1,2,\cdots,r)$: IF $z_1(t)$ is M_{i1}, $z_2(t)$ is $M_{i2},\cdots$, and $z_p(t)$ is M_{ip}. THEN $\dot{x}(t)=A_ix(t)$, where r is no. of T-S fuzzy rules, $z_1(t)$, $z_2,\cdots$, $z_p(t)$ are the premise variables, each M_{ij} is a fuzzy set and $A_i\subseteq R^{n*n}$ is a constant matrix. Thus, the non-linear equations can be transformed into the following linear equation.

If x(t) is M_i then

$$\dot{x}(t)=A_ix(t), \qquad t\neq\tau_j, \qquad i=1,2,3,\cdots,r; \quad j=1,2,\cdots, \tag{3.1a}$$

$$\Delta(x)=K_{ij}x(t), \qquad t=\tau_j, \qquad i=1,2,3,\cdots,r; \quad j=1,2,\cdots, \tag{3.1b}$$

where

$$A_i=\begin{bmatrix} r-ex-\frac{P_1y_s}{a_0+x}-\frac{P_2y_i}{a_0+x}-\frac{P_3z}{a_0+x} & 0 & 0 & 0 \\ \frac{C_1P_1y_s}{a_0+x} & -\frac{C_1P_1z}{a_1+x}-\beta y_i-m_1 & 0 & 0 \\ \frac{C_2P_2y_i}{a_0+x} & \beta y_i & -\frac{C_2P_2z}{a_2+x}-m_2 & 0 \\ \frac{C_3P_3z}{a_0+x} & -\frac{C_3P_3z}{a_1+x} & \frac{C_3P_3z}{a_2+x} & -m_3 \end{bmatrix}$$

and z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8, z_9 are related to the values of $x(t)$, $y_s(t)$, $y_i(t)$, $z(t)$ (here $z_1=ex$, $z_2=\frac{P_1y_s}{a_0+x}$, $z_3=\frac{P_2y_i}{a_0+x}$, $z_4=\frac{P_3z}{a_0+x}$, $z_5=\frac{P_1z}{a_1+x}$, $z_6=\beta y_i$, $z_7=\frac{P_2z}{a_2+x}$, $z_8=\frac{P_3z}{a_1+x}$, $z_9=\frac{P_3z}{a_2+x}$). M_i, x(t), $A_i\in R^{4*4}$, r is the number of the IF-THEN rules, $K_{i,j}$ denotes the control of the j^{th} impulsive instant,

$$\Delta(x)|_{t=\tau_j}=x(\tau_j-\tau_{j-1}).$$

With center-average de-fuzifier, the overall T-S fuzzy impulsive system can be represented as:

$$\dot{x}(t)=\sum_{i=1}^{r}h_i(z(t))(A_ix(t)), \qquad t\neq\tau_j, \tag{3.2a}$$

$$\Delta(x)=\sum_{i=1}^{r}h_i(z(t))K_{ij}, \qquad t=\tau_j, \tag{3.2b}$$

where

$$h_i(z(t))=\omega_i(z(t))\Big/\sum_{i=1}^{r}\omega_i(z(t)) \quad \text{and} \quad \omega_i(z(t))=\prod_{j=1}^{p}M_{ij}(z(t)).$$

Evidently, $h_i(z(t))\geq 0$, $\sum_{i=1}^{r}h_i(z(t))=1$, $i=1,2,\cdots,r$.

4 Stability analysis

We shall now analyse several stability properties of the impulsive fuzzy system 3.2 by considering the following theorems [45].

Theorem 4.1. *Assume that λ_i is the maximum eigen value of $[A_i^T+A_i]$, $(i=1,2,3,\cdots,r)$. Let $\lambda(\alpha)=\max_i\{\lambda_i\}$, $0<\delta_j=\tau_j-\tau_{j-1}<\infty$ is the impulsive distance. If $\lambda(\alpha)\geq 0$ and there exists a constant scalar $\epsilon>1$ and a semi-positive matrix P, such that*

$$\ln(\epsilon\beta_j)+\lambda(\alpha)\delta_j\leq 0, \quad PA_i=A_iP, \tag{4.1}$$

where

$$P=C^TC, \quad \beta_j=\max\|C(I+K_{i,j})\|, \tag{4.2}$$

then the system (3.2) *is stable globally and asymptotically.*

Theorem 4.2. *Assume that λ_i is the maximum eigen value of $[A_i^T+A_i]$, $(i=1,2,3,\cdots,r)$. Let $\lambda(\alpha)=\max_i\{\lambda_i\}$, $0<\delta_j=\tau_j-\tau_{j-1}<\infty$ is the impulsive distance. If $\lambda(\alpha)<0$ and a constant scalar $0\leq\epsilon<-\lambda(\alpha)$, such that*

$$\ln(\beta)-\epsilon\delta_j\leq 0, \quad PA_i=A_iP, \tag{4.3}$$

where

$$\beta_j=\max\|C(I+K_{ij})\|, \quad P=C^TC, \tag{4.4}$$

then the system (3.2) *is stable globally and exponentially.*

5 Numerical simulation

Since most of the biological systems are complex, they should be modelled using an expressive description and fuzzy logic. Consequently, the suggested impulsive T-S design model examines predator-prey systems with functional response and impulsive effects.

By using fuzzy impulsive T-S design model on (2.2), the membership functions [10] obtained as

$$
\begin{aligned}
&M_1=\frac{z_1}{ed_1}, && M_2=\frac{ed_1-z_1}{ed_1}, && N_1=\frac{z_2}{\frac{P_1d_2}{a_0+d_1}}, && N_2=\frac{\frac{P_1d_2}{(a_0+d_1)}-z_2}{\frac{P_1d_2}{(a_0+d_1)}},\\
&K_1=\frac{z_3}{\frac{P_2d_3}{a_0+d_1}}, && K_2=\frac{\frac{P_2d_3}{a_0+d_1}-z_3}{\frac{P_2d_3}{a_0+d_1}}, && L_1=\frac{z_4}{\frac{P_3d_4}{a_0+d_1}}, && L_2=\frac{\frac{P_3d_4}{a_0+d_1}-z_4}{\frac{P_3d_4}{a_+d_1}},\\
&O_1=\frac{z_5}{\frac{P_1d_4}{a_1+d_1}}, && O_2=\frac{\frac{P_1d_4}{a_1+d_1}-z_5}{\frac{P_1d_4}{a_1+d_1}}, && R_1=\frac{z_6}{\beta d_3}, && R_2=\frac{\beta d_3-z_6}{\beta d_3},\\
&S_1=\frac{z_7}{\frac{P_2d_4}{a_2+d_1}}, && S_2=\frac{\frac{P_2d_4}{a_2+d_1}-z_7}{\frac{P_2d_4}{a_2+d_1}}, && T_1=\frac{z_8}{\frac{P_3d_4}{a_1+d_1}}, && T_2=\frac{\frac{P_3d_4}{a_1+d_1}-z_8}{\frac{P_3d_4}{a_1+d_1}},\\
&P_1=\frac{z_9}{\frac{P_3d_4}{a_2+d_1}}, && P_2=\frac{\frac{P_3d_4}{a_2+d_1}-z_9}{\frac{P_3d_4}{a_2+d_1}},
\end{aligned}
$$

and the matrices $A_i's$ are calculated using

$$
A_i=\begin{bmatrix}
r-ex-\dfrac{P_1y_s}{a_0+x}-\dfrac{P_2y_i}{a_0+x}-\frac{P_3z}{a_0+x} & 0 & 0 & 0\\
\dfrac{C_1P_1y_s}{a_0+x} & -\dfrac{C_1P_1z}{a_1+x}-\beta y_i-m_1 & 0 & 0\\
\dfrac{C_2P_2y_i}{a_0+x} & \beta y_i & -\dfrac{C_2P_2z}{a_2+x}-m_2 & 0\\
\dfrac{C_3P_3z}{a_0+x} & -\dfrac{C_3P_3z}{a_1+x} & \dfrac{C_3P_3z}{a_2+x} & -m_3
\end{bmatrix},
$$

$i=1$ to 511, and, the Defuzzification can be represented as:

$$
\dot{x}(t)=\sum_{i=1}^{r}h_i(z(t))(A_ix(t)), \tag{5.1}
$$

here $h_i's$ are given as

$$
h_i(z(t))=\omega_i(z(t))\Big/\sum_{i=1}^{r}\omega_i(z(t)) \quad \text{and} \quad \omega_i(z(t))=\prod_{j=1}^{p}M_{ij}(z(t)),
$$

$i=1$ to 511, $j=1$ to 9. This Fuzzy model exactly represents the non-linear system in the region $[0,5]\times[0,10]\times[0,10]$.

$$\frac{dx}{dt}=rx-ex^2-\frac{P_1y_sx}{a_0+x}-\frac{P_2y_ix}{a_0+x}-\frac{P_3zx}{a_0+x}, \tag{5.2a}$$

$$\frac{dy_s}{dt}=\frac{C_1P_1y_sx}{a_0+x}-\frac{C_1P_1y_sz}{a_1+x}-\beta y_sy_i-m_1y_s, \tag{5.2b}$$

$$\frac{dy_i}{dt}=\beta y_sy_i+\frac{C_2P_2y_ix}{a_0+x}-\frac{C_2P_2y_iz}{a_2+x}-m_2y_i, \tag{5.2c}$$

$$\frac{dz}{dt}=\frac{C_3P_3xz}{a_0+x}-\frac{C_3P_3y_sz}{a_1+x}+\frac{C_3P_3y_iz}{a_2+x}-m_3z. \tag{5.2d}$$

6 Results and discussion

In this section, the global stability of the considered intra-specific competition predator-prey model (2.2) is discussed. The fuzzy logical technique with linguistic description should be used to depict biological systems because they are complicated, nonlinear, and unpredictable.

Calculations were carried by taking the values of the parameters at $r=1.5$, $e=0.2$, $\beta=0.4$, $P_1=0.7$, $P_2=0.06$, $P_3=0.8$, $C_1=0.5$, $C_2=0.4$, $C_3=0.36$, $m_1=0.1$, $m_2=0.5$, $m_3=0.4$, $a_0=1.0$, $a_1=1.0$, $a_2=1.0$, $\mathcal{R}_0=0.8$, $d_1=10$, $d_2=10$, $d_3=10$, $d_4=10$ in 3.1 to get the eigen values of $[A_i^T+A_i](i=1,2,3,\cdots,r)$ as explained in the Theorems 4.1,4.2. It is found that $\max(\lambda_i)=\lambda(\alpha)=1.5$, then we have chosen $diag[-0.99,-0.99]$ as impulsive control matrix, such that $\beta_j=\|I+K\|=0.01$. It is noted that the system 3.1 is stable globally (4.1) when ϵ=1.5, δ_j=0.1 (at those above values, $\ln(\epsilon\beta_j)+\lambda(\alpha)\delta_j=-4.184<0$). Further, it is observed that the prey-predator model is unstable (4.1) when $r=42$, $e=5.5$, $\beta=5.5$, $P_1=2.8$, $P_2=2.8$, $P_3=2.0$, $C_1=1.8$, $C_2=2.2$, $C_3=5.6$, $m_1=5.2$, $m_2=5.4$, $m_3=0.5$, $a_0=1.0$, $a_1=1.0$, $a_2=1.0$, $\mathcal{R}_0=1.01$, $d_1=10$, $d_2=10$, $d_3=10$, $d_4=10$, since

$$\max(\lambda_i)=\lambda(\alpha)=42 \implies \ln(\epsilon\beta_j)+\lambda(\alpha)\delta_j=0.001>0$$

for $\beta_j=0.01$, $\epsilon=1.5$, $\delta_j=0.1$. Table 1 presents the stability of the system at various values of the present study.

Table 1: Stability analysis by taking different values of the parameters.

r	e	β	P_1	P_2	P_3	C_1	C_2	C_3	m_1	m_2	m_3	a_0	a_1	a_2	d_1	d_2	d_3	d_4	$\max(\lambda_i)$ $\lambda(\alpha)$	$\ln(\epsilon\beta)$ $+\lambda(\alpha)\delta_j$	conclusions
1.5	0.2	0.4	0.7	0.06	0.8	0.5	0.4	0.36	0.1	0.5	0.4	1.0	1.0	1.0	10.0	10.0	10.0	10.0	1.5	-4.184	stable
2.0	0.5	0.5	0.8	0.6	1.0	0.5	0.4	0.6	0.5	0.2	0.5	1.0	1.0	1.0	10.0	10.0	10.0	10.0	2.0	-3.999	stable
2.5	1.5	0.2	1.8	1.6	2.0	0.8	0.2	0.6	0.2	0.4	0.5	1.0	1.0	1.0	10.0	10.0	10.0	10.0	2.5	-3.949	stable
42	5.5	5.5	2.8	2.8	2.0	1.8	2.2	5.6	5.2	5.4	0.5	1.0	1.0	1.0	10.0	10.0	10.0	10.0	42	0.001	unstable

The impact of the some parameters on Prey-predator system 2.1 with T-S fuzzy impulsive control model is presented in Figs. 1-10 by fixing few parameters $P_1=0.7$, $P_2=0.06$, $P_3=0.8$, $C_1=0.5$, $C_2=0.4286$, $C_3=0.36$, $a_0=1.0$, $a_1=1.0$, $a_2=1.0$, $m_1=0.1$. The dynamical change on prey-predator population (x,y) by varying intrinsic growth rate of prey (r) parameter under fuzzy impulsive control can be noted in Fig. 1 at $e=0.2$, $\beta=0.7$, $m_2=0.5$, $m_3=0.4$, $d_1=10$, $d_2=10$, $d_3=10$, $d_4=10$. It is observed from this figure that, increase in r increases population of prey.

The effectiveness by varying intra-specific competition (e) parameter of prey-predator population (x,y) under fuzzy impulsive control can be noted in Fig. 2 at $r=1.5$, $\beta=0.7$, $m_2=0.5$, $m_3=0.4$, $d_1=10$, $d_2=10$, $d_3=10$, $d_4=10$. This figure clearly shows that increase in e decreases population of prey, but increases infected first predator and second predator population. The influence of prey max time d_1

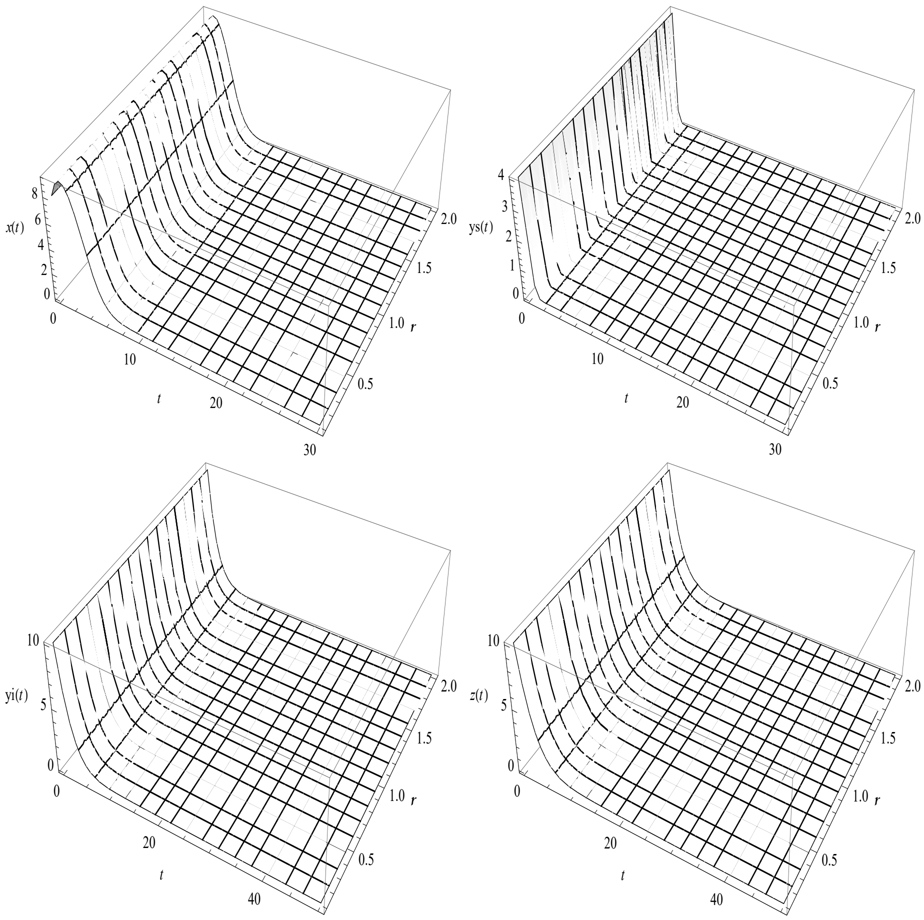

Figure 1: Phase portrait figure showing the effect of intrinsic growth rate of prey (r) parameter on the prey-predator system under impulsive control.

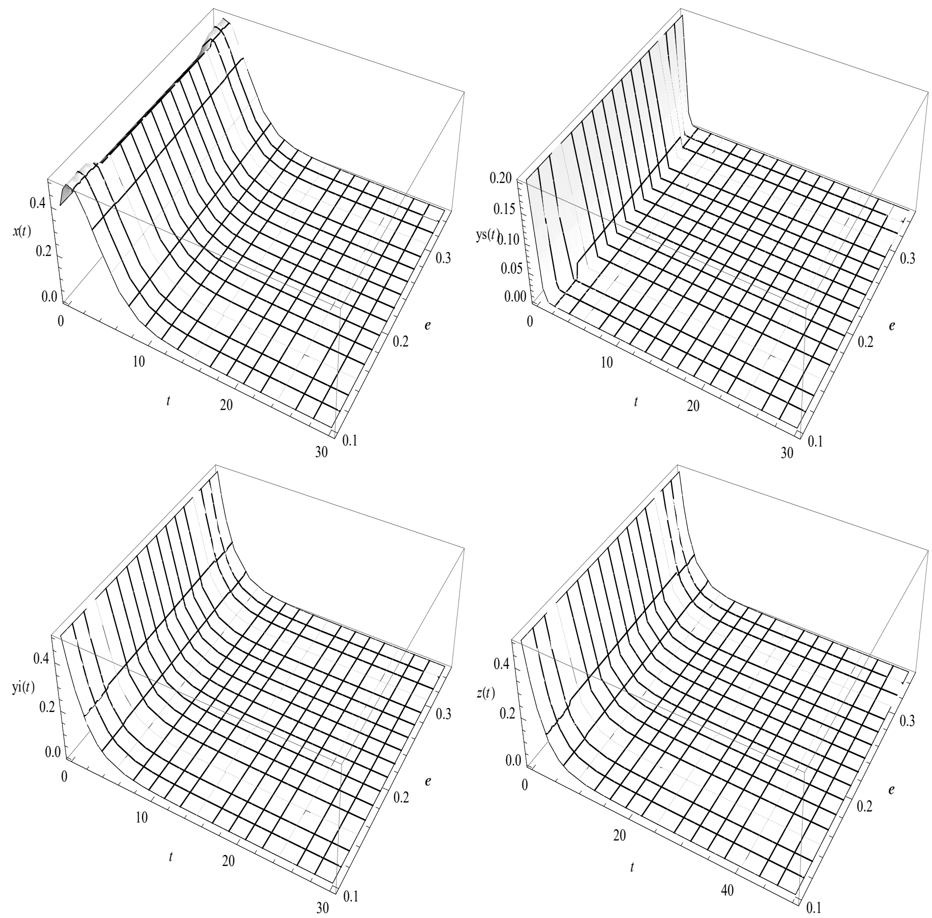

Figure 2: Phase portrait figure showing the effect of intra-specific competition (e) parameter on the prey-predator system under impulsive control.

on prey-predator system is shown in Fig. 3 at $r=1.5$, $e=0.2$, $\beta=0.7$, $m_2=0.5$, $m_3=0.4$, $d_2=10$, $d_3=10$, $d_4=10$. This graph makes it abundantly evident that as d_1 increases, population increases for prey and first predator but decreases for second predator.

The influence of susceptible first predator max time d_2 on prey-predator system is shown in Fig. 4 at $r=1.5$, $e=0.2$, $\beta=0.7$, $m_2=0.5$, $m_3=0.4$, $d_1=10$, $d_3=10$, $d_4=10$. This graph shows that as d_2 increases, prey population decreases but predators population increases. The change on prey-predator system with max time of infected first predator (d_3) is shown in Fig. 5 at $r=1.5$, $e=0.2$, $\beta=0.7$, $m_2=0.5$, $m_3=0.4$, $d_1=10$, $d_2=10$, $d_4=10$. This figure clearly exhibits that as d_3 increases, population decreases for prey and susceptible first predator.

The outcome with varying max time of second predator (d_4) on prey-predator

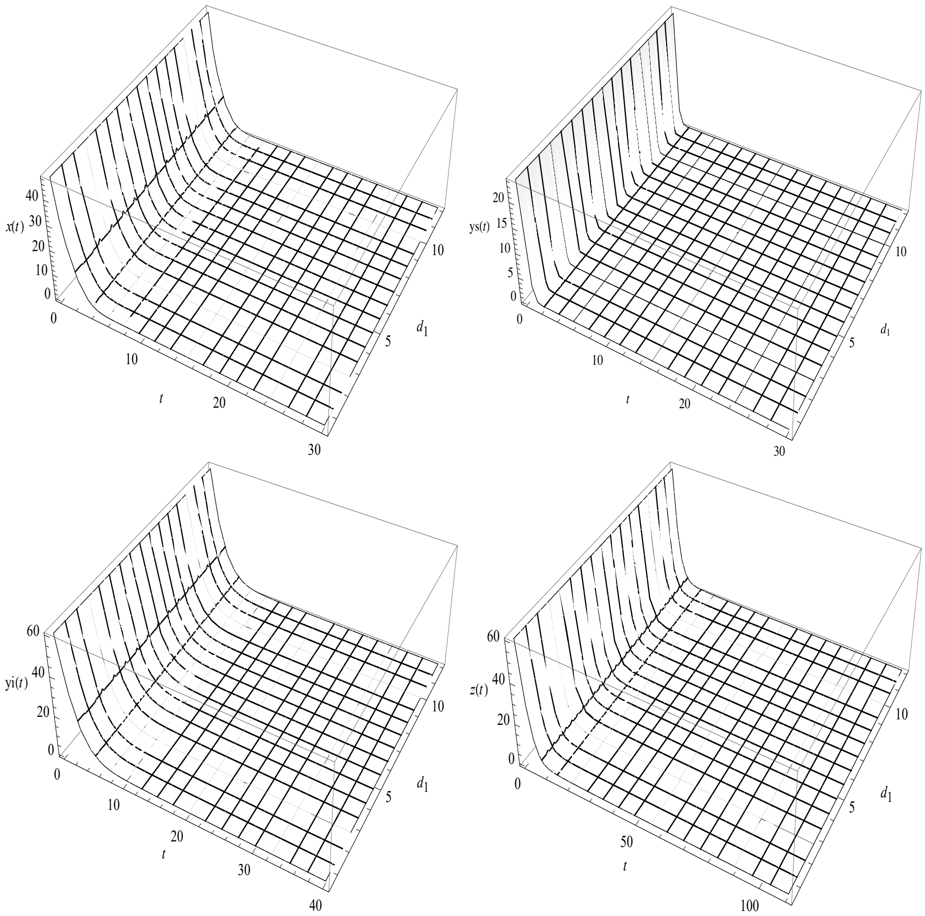

Figure 3: Phase portrait figure showing the effect of max time of prey (d_1) parameter on the prey-predator system under impulsive control.

system is shown in Fig. 6 at $r=1.5$, $e=0.2$, $\beta=0.7$, $m_2=0.5$, $m_3=0.4$, $d_1=10$, $d_2=10$, $d_3=10$. This graph illustrates clearly how increase in second predator max time decreases prey population and first predator population but second predator population increases.

The effect of transmission coefficient from susceptible first predator to infected first predator parameter β on prey-predator system is shown in Fig. 7 at $r=1.5$, $e=0.2$, $m_2=0.5$, $m_3=0.4$, $d_1=10$, $d_2=10$, $d_3=10$, $d_4=10$. This graph shows that as transmission coefficient from susceptible first predator to infected first predator rise, the population of susceptible first predator decreases.

The vital pattern of prey- predator population (x,y) by varying mortality rate of infected first predator (m_2) parameter under fuzzy impulsive control can be noted in Fig. 8 at $r=1.5$, $e=0.2$, $\beta=0.7$, $m_3=0.4$, $d_1=10$, $d_2=10$, $d_3=10$, $d_4=10$. This

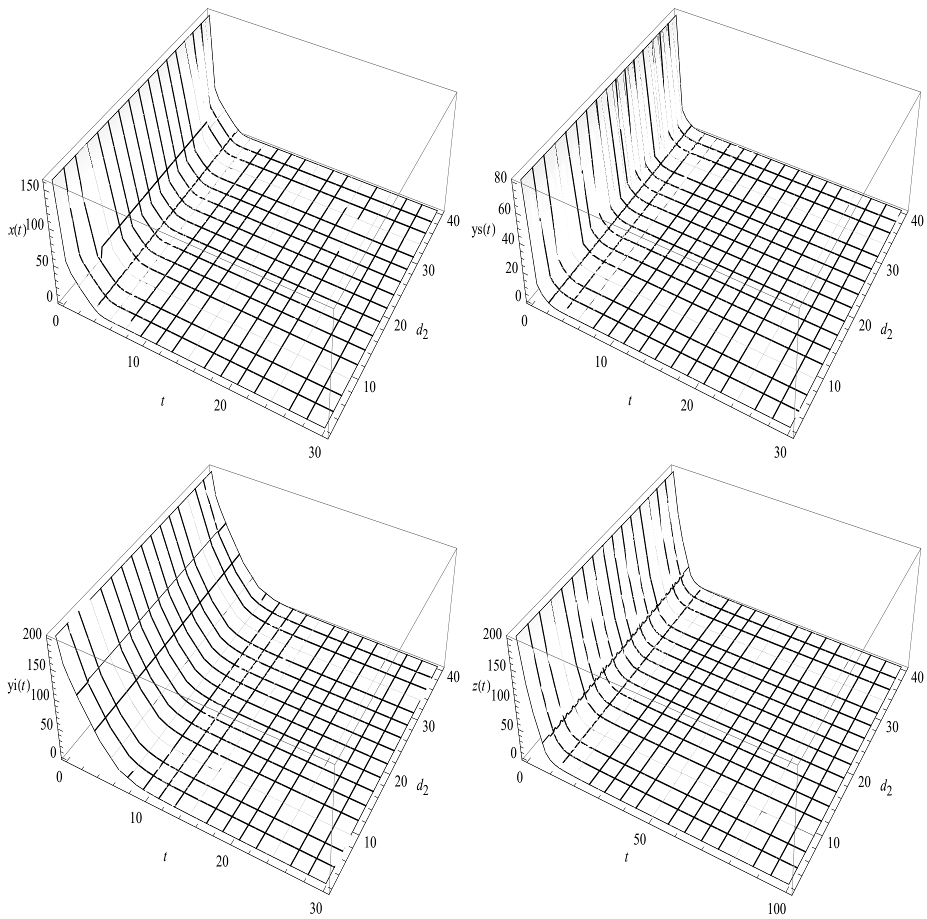

Figure 4: Phase portrait figure showing the effect of max time of susceptible first predator (d_2) parameter on the prey-predator system under impulsive control.

figure clearly exhibits that as m_2 increases, population of susceptible and infected first predator decreases.

The change on prey-predator system (x,y) by varying mortality rate of second predator (m_3) parameter under fuzzy impulsive control can be noted in Fig. 9 at $r=1.5$, $e=0.2$, $\beta=0.7$, $m_2=0.5$, $d_1=10$, $d_2=10$, $d_3=10$, $d_4=10$. This figure clearly exhibits that as m_3 increases, population of prey and second predator decreases but population of infected first predator increases.

Finally, the nature of prey-predator system without impulsive control is presented in Fig. 10 by fixing all the parameters obtained from T-S fuzzy model at $r=1.5$, $e=0.2$, $\beta=0.7$, $P_1=0.7$, $P_2=0.06$, $P_3=0.8$, $C_1=0.5$, $C_2=0.4$, $C_3=0.36$, $m_1=0.1$, $m_2=0.5$, $m_3=0.4$, $a_0=1.0$, $a_1=1.0$, $a_2=1.0$, $d_1=10$, $d_2=10$, $d_3=10$, $d_4=10$, and initial conditions $x_s(0)>0$, $x_i(0)>0$, $y(0)>0$, $z(0)>0$, $t=10$. The figure clearly

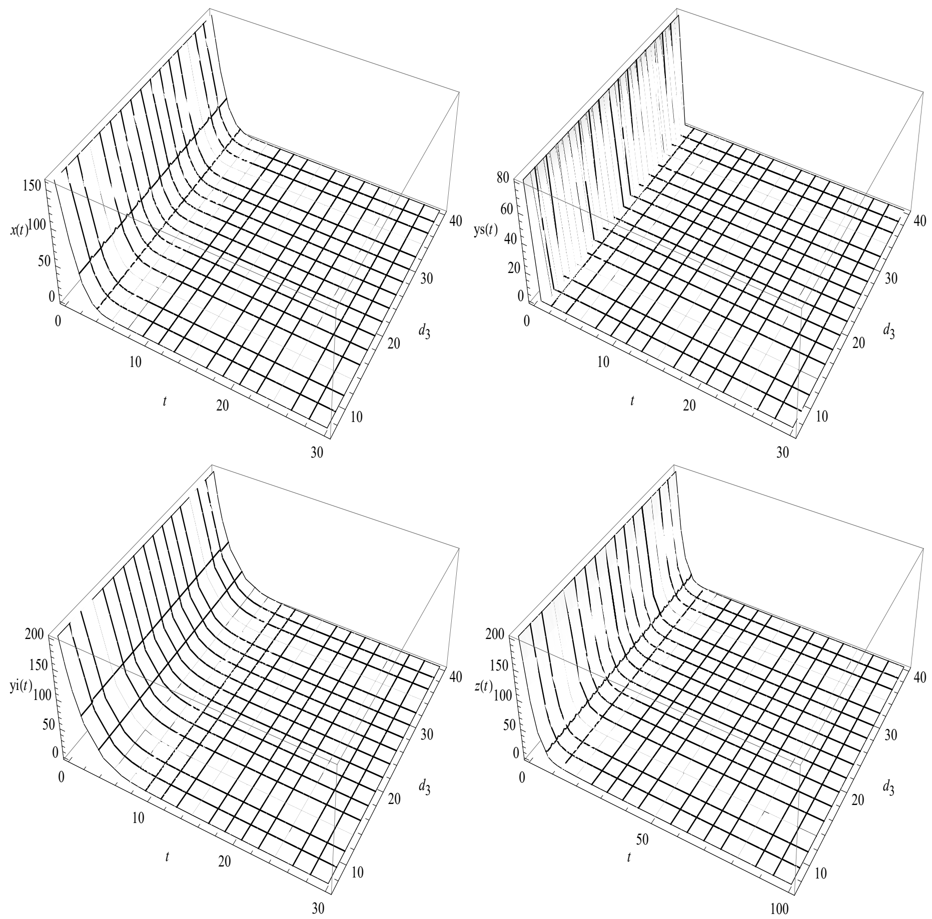

Figure 5: Phase portrait figure showing the effect of max time of infected first predator (d_3) parameter on the prey-predator system under impulsive control.

shows how the prey and predator populations reaches to stability.

7 Conclusions

In many disciplines, including ecology, dynamics, physics, algorithms, and epidemiology, mathematical models are crucial. In this work, a predator-prey model with two predator populations and one prey population is built, but only the first predator population is infected. First, a fuzzy impulsive control-based non-linear Lotka-Volterra predator-prey model was examined. The fuzzy systems based on the T-S model are used to examine the impulsive control technique, which is found to be suitable for extremely complex non-linear systems with impulsive effects. Addition-

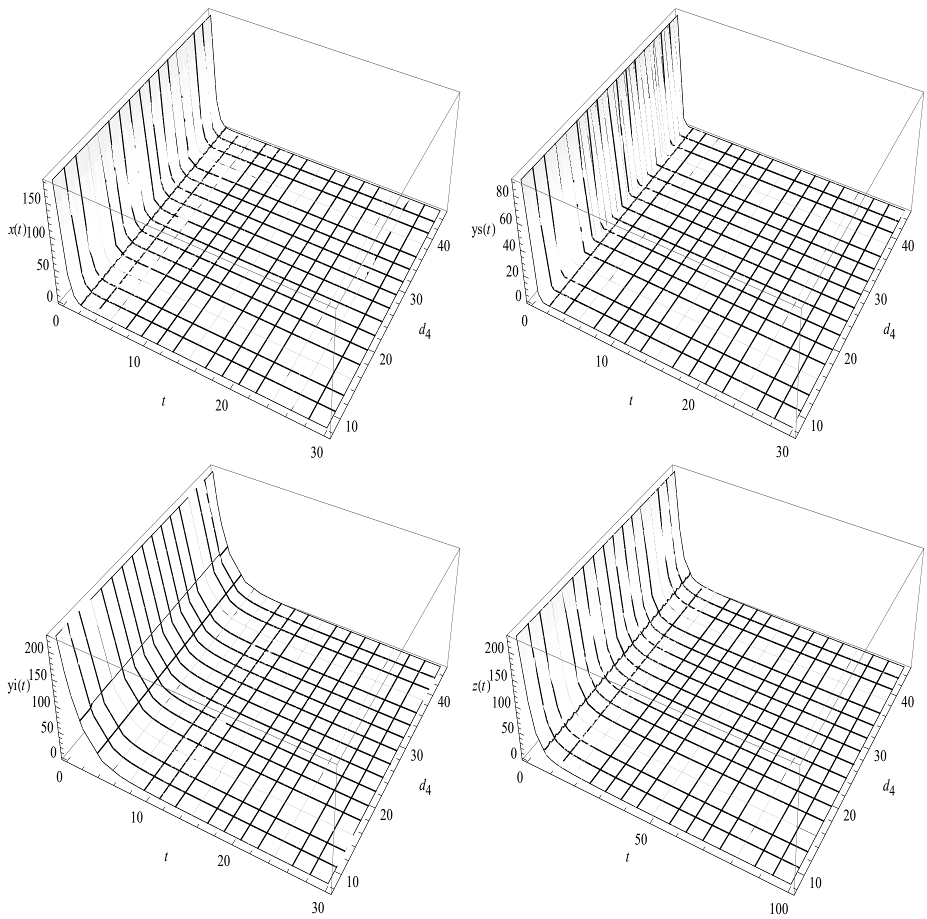

Figure 6: Phase portrait figure showing the effect of max time of second predator (d_4) parameter on the prey-predator system under impulsive control.

ally, each local linear impulsive system is combined to create the full impulsive fuzzy system. In the meantime, numerous stability theorems demonstrate the impulsive fuzzy system's asymptotic stability and exponential stability. Finally, to illustrate the usage of impulsive fuzzy control, a numerical example of predator-prey systems with impulsive effects is shown. Simulation results show the value of the suggested method. According to the references already in existence, the current investigation covers a variety of ecological consequences and got adequate results.

- Intrinsic growth rate of prey effects all the four populations. Increase in intrinsic growth rate of prey increases population of prey, and decreases population of susceptible first predator, infected first predator and second predator.

- Increase in intra-specific competition decreases population of prey, but popu-

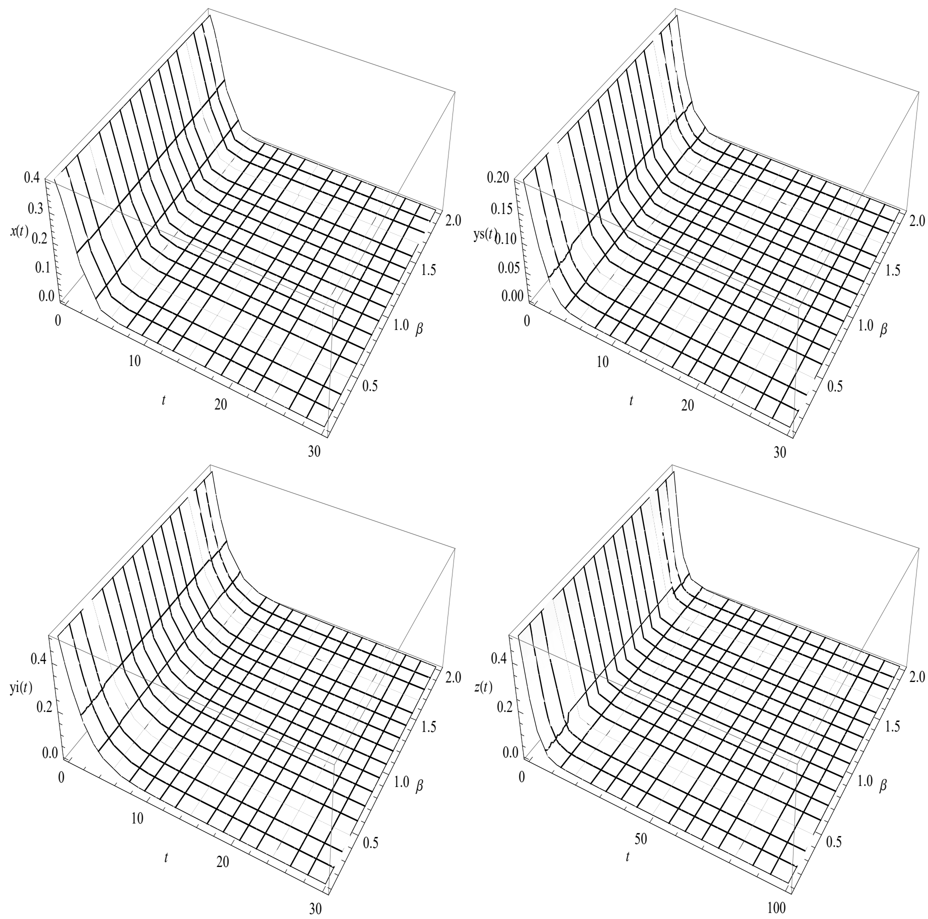

Figure 7: Phase portrait figure showing the effect of transmission coefficient from susceptible first predator to infected first predator (β) parameter on the prey-predator system under impulsive control.

lation of infected first predator and second predator increases.

- As the mortality rate of infected first predator increases, population of susceptible first predator and infected first predator decreases.

- Increase in the mortality rate of second predator, decreases population of prey and second predator.

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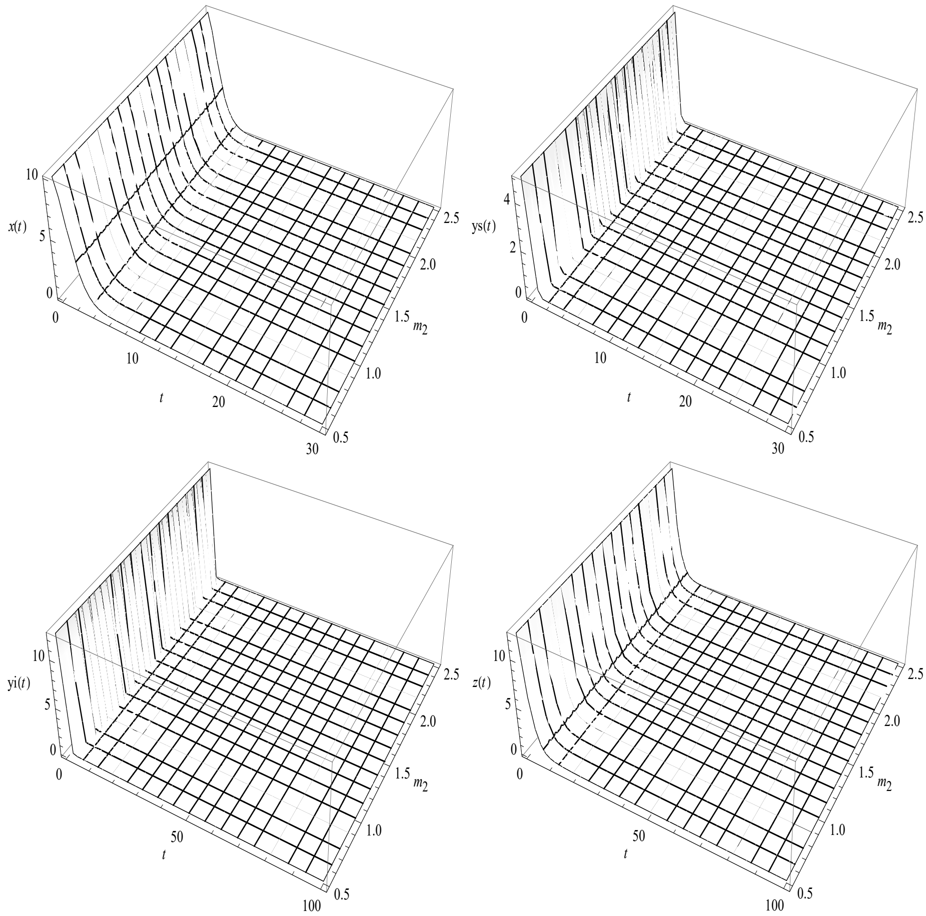

Figure 8: Phase portrait figure showing the effect of mortality rate of infected first predator (m_2) parameter on the prey-predator system under impulsive control.

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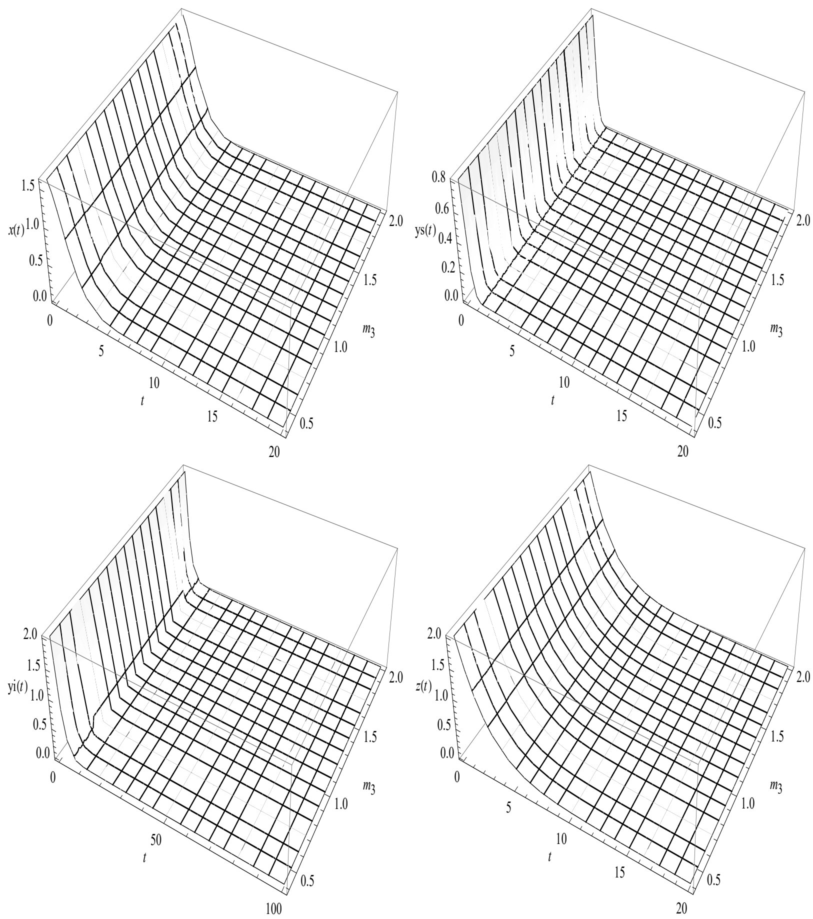

Figure 9: Phase portrait figure showing the effect of mortality rate of second predator (m_3) parameter on the prey-predator system under impulsive control.

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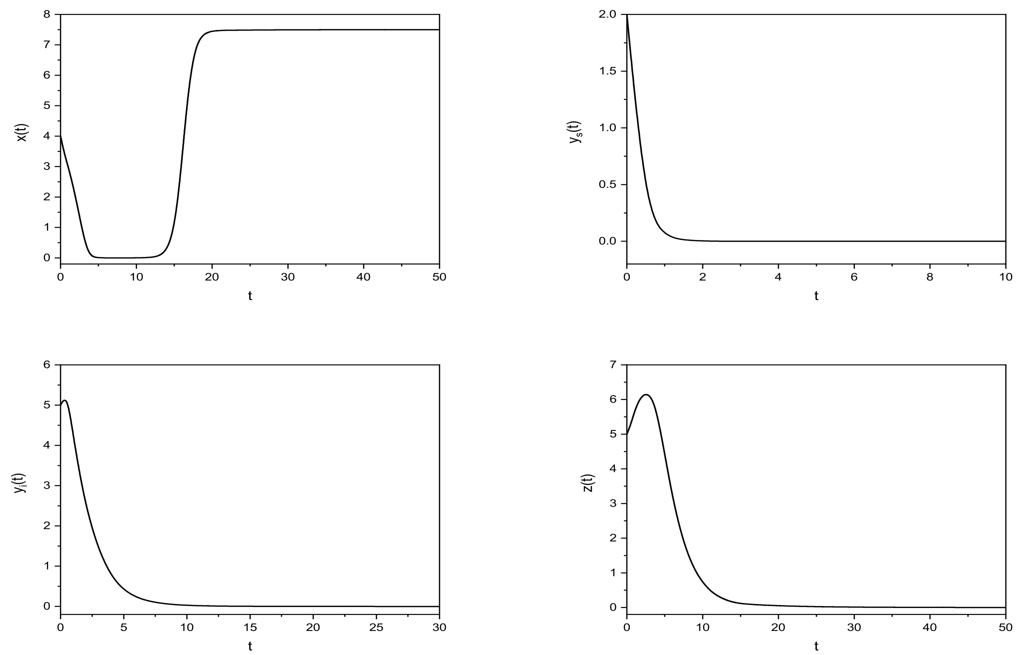

Figure 10: Plot of predator-prey system without impulsive control.

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