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A Four-Equation Eddy-Viscosity Approach for Modeling Bypass Transition

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Abstract. It is very important to predict the bypass transition in the simulation of flows through turbomachinery. This paper presents a four-equation eddy-viscosity turbulence transition model for prediction of bypass transition. It is based on the SST turbulence model and the laminar kinetic energy concept. A transport equation for the non-turbulent viscosity is proposed to predict the development of the laminar kinetic energy in the pre-transitional boundary layer flow which has been observed in experiments. The turbulence breakdown process is then captured with an intermittency transport equation in the transitional region. The performance of this new transition model is validated through the experimental cases of T3AM, T3A and T3B. Results in this paper show that the new transition model can reach good agreement in predicting bypass transition, and is compatible with modern CFD software by using local variables.

AMS subject classifications: 76F06, 76F55

Key words: Bypass transition, non-turbulent viscosity, free stream turbulence, turbulence model.

1 Introduction

Laminar to turbulence transition is an important issue in modern fluid mechanics. Numerous theoretical and experimental studies on the incompressible boundary layer flow over a smooth flat plate are available in the literature. In general, two different transition routes are identified for the zero pressure gradient flat plate: natural and bypass transitions. The Tollmien-Schlichting (T-S) instability waves will lead to turbulence on flat pate in the very low levels of free-stream turbulent environment. But the T-S waves will be bypassed and a rapid transition process occurs when free-stream turbulence intensity

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exceeds 1% of the free-stream velocity. In the past, bypass transition has been extensively studied for its importance in engineering design. Dryden [1] and Taylor [2] were probably the first in conducting the experimental studies of transition under the influence of intense free-stream turbulence. They observed span-wise alternating low and high-speed regions in the pre-transitional boundary layer flow. Klebanoff [3] showed that disturbances in the low and high-speed regions grow more or less linearly with the boundary layer thickness. Kendall [4, 5] showed that disturbances are low-frequency and streaky fluctuations and originate from the leading edge, and Kendall figured out that the disturbances are not turbulence. Similar to the work of Klebanoff [3], Westin et al. [6] found that the stream-wise velocity disturbance varies linearly with the square root of the distance from the beginning of the leading edge. Matsubare & Alfredsson [7], in their experimental studies, showed that free-stream turbulence penetrates the boundary layer and then induces the stream-wise velocities in the pre-transition region of the boundary layer flow. They also pointed out that the span-wise wavelength of streaky fluctuation is of the order of the boundary layer thickness. Fransson, Matsubare & Alfredsson [8] studied bypass transition which focused on the modeling of the transition zone under different conditions. All the results above confirm that disturbances in the region of boundary layer flow are low-frequency non-turbulent fluctuation whose streamwise velocity has a much larger magnitude than both the normal and spanwise velocities.

Bypass transition has also been studied by direct numerical simulation (DNS) and Large eddy simulation (LES). In the process, the mathematical modeling of the non-turbulent fluctuation is added to DNS or LES as the inlet condition. The free stream turbulence is commonly modeled with the continuous spectra of the Orr-Sommerfeld (OS) and Squire operators in most of the research work (Jacobs & Durbin [9], Zaki & Durbin [10] and Yang Liu, Tamer A. Zaki & Paul A. Durbin [11]). Butler & Farrell [12], Andersson, Berggren & Henningson [13] and Luchini [14] derived a different model based on non-modal growth analyses approach. These mathematical models and DNS/LES methods are very useful for academic simulations, but are very costly and expensive for engineering applications.

Almost all experimental results reveal that the flow becomes intermittent in the transition region when the bypass transition occurs. Dhawan & Narasimha [15] first considered transition as an eruption of turbulent spot. The physical nature of the transition flow can be described with the intermittent factor. There are many algebraic models for intermittency based on this idea and experimental data in previous works [16]. But most algebraic models are valid only for the flow with zero pressure gradient and TS wave transition.

A further approach into intermittency modeling is obtained as a solution of the intermittency transport equation [17–24]. The main advantage of this approach is that the modeling of the transition process is not only in the flow direction but also across the boundary layer, and hence provides a more realistic prediction method [22]. But most intermittency transport equations require non-local information which is not easy to obtain in CFD solvers, such as the boundary layer thickness and the free-stream velocity. And the transition onset must be required for the intermittency transport equation with some empirical correlations such as Mayle correlation [25] and Abu-Ghannam & Shaw correlation [26]. However, the $\gamma - Re_\theta$ transition model introduced by Menter et al. [27] overcomes this shortcoming and is appropriate to simulate flows around complex geometries and has a great potential to be applied to practical problems. It has been validated that the turbulence transition in subsonic 2-D boundary layer flows can be predicted with Menter et al.'s model, but there is a lack of the fundamental physical phenomena responsible for transition process.

As mentioned above, the disturbances in the region of boundary layer flow are low-frequency non-turbulent fluctuation. The low frequency non-turbulent fluctuation is defined as "laminar kinetic energy". A novel and interesting class of transition models is based on the concept and the modeling of the laminar kinetic energy in the pre-transitional boundary layer region. Mayle & Schulz [28] firstly proposed a transport equation for the laminar kinetic energy. Walters & Leylek [29] adopted Mayle & Schulz's [28] idea and developed a locally formulated transport equation for the laminar kinetic energy that describes the development of the non-turbulent streamwise fluctuations in the pre-transitional boundary layer region, but Walters & Leylek' transition model has too many model constants. In order to model of the high-speed flow transition, Wang & Fu [30] developed a $k-\omega-\gamma$ three-equation approach that is capable of accommodating the effect of the second instability mode, while non-local parameter is required in Wang & Fu's transition model such as the local mean velocity at the generalized inflection point.

However, there is still some space for further improvements in the modeling of transition flows as the transition process is highly nonlinear and highly complicated. The present work develops a four-equation approach for bypass transition modeling. It is based on the SST turbulence model and the concept of laminar kinetic energy. A locally formulated transport equation for the non-turbulent viscosity, rather than the laminar kinetic energy equation, is introduced to describe the development of the streak in the pre-transitional boundary layer. A new intermittency transport equation is also coupled to capture the transition process and establishes a link between the laminar and the turbulent flows.

2 A four-equation bypass turbulence transition model

2.1 Transport equation for the non-turbulent viscosity (streaky fluctuations)

As mentioned above, the non-turbulent (streaky fluctuations) and turbulent fluctuations coexist in the transition region, with the non-turbulent fluctuation in the form of streak disturbance in the bypass transition. Based on Lin's analysis [31] for unsteady laminar boundary layers and Dullenkopf & Mayle's concept [32] of an effective frequency and turbulence level for laminar boundary layers, Mayle & Schulz [28] proposed the laminar-kinetic-energy (LKE) transport equation to predict the non-turbulent fluctuations, and the source term (or the production term) in the transport equation is argued to arise from the pressure-diffusion correlation. However, Lardeau et al. [33] showed in their LES simulation that the pressure-diffusion term in Mayle & Schulz's equation is actually a sink over most part of the flow and the elevation of the fluctuation energy is due to the shear production rather than pressure-diffusion, despite that the level of the shear stress is relatively low. Similar to the modeling of the laminar kinetic energy, the effective viscosity of the non-turbulent fluctuation has been developed by Fu & Wang [23] in an algebraic manner. However, it should be indicated that the evolution and spatial distribution of the non-turbulent fluctuation cannot be accounted for in the algebraic turbulence transition model.

In this paper, a transport equation for the kinematic viscosity of non-turbulent fluctuation has been developed by the authors as follows:

$$\frac{\partial(\rho\nu_L)}{\partial t}+\frac{\partial(\rho U_i\nu_L)}{\partial x_i}=P_{bypass}-\varepsilon_{\nu_L}+\frac{\partial}{\partial x_i}\Big[\rho(\nu+\sigma_{L1}\nu_L+\sigma_{L2}\nu_T)\frac{\partial \nu_L}{\partial x_i}\Big]. \tag{2.1}$$

The non-turbulent kinematic viscosity is defined as ν_L in this paper, ν and ν_T denote the laminar and turbulent kinematic viscosity respectively. Based on Lardeau et al.'s work [33] that the kinetic energy of non-turbulence fluctuation is produced by the shear stress of boundary layer flow, the production term for the non-turbulent kinematic viscosity is defined as follows:

$$P_{bypass}=f_{FST}\rho u_m\zeta S, \tag{2.2}$$

where the u_m and ζ represent the velocity and length scales of the non-turbulent fluctuation, respectively, and S is the strain rate magnitude. In the transition region, the total disturbance energy k is a combination of the non-turbulent fluctuation energy k_L and the turbulent fluctuation energy k_t, that is,

$$k=k_L+k_t,\quad k_L=(1-\gamma)k,\quad k_t=\gamma k.$$

So the velocity scale of the non-turbulent fluctuation is defined as $u_m=\sqrt{(1-\gamma)k}$ and γ is the intermittency factor. Similar to Fu & Wang [23], the length scale of non-turbulent fluctuation is defined as follows:

$$\zeta=\Omega d^2/(2E_u)^{0.5}.$$

Here, d is the distance to wall, Ω is the absolute value of the mean vorticity, and E_u stands for the disturbance kinetic energy of the mean flow related to the wall. In order to model the bypass transition, the effect of free stream turbulence must be taken into account. As such, the form of f_{FST} used is

$$f_{FST}=C_1\gamma\sqrt{1-\gamma}\Big\{1-\exp\Big[-C_2\Big(\frac{k}{S^2\zeta^2}\Big)^{C_3}\Big]\Big\}\frac{\Omega\zeta^2}{\nu}.$$

The production term of non-turbulent viscosity will be zero in the laminar $(\gamma=0)$ and turbulent region $(\gamma=1)$ due to the function $\gamma\sqrt{1-\gamma}$, respectively, and the term $\Omega\zeta^2/\nu$ indicates that the non-turbulent viscosity will not grow in the presence of irrotational straining. The term $1-\exp\{-C_2[k/(S^2\zeta^2)]^{C_3}\}$ is a damping function which is used to account for the effect of free stream turbulence [29]. Thus, the production term of the non-turbulent fluctuation is obtained as follows:

$$P_{bypass}=C_1\gamma\sqrt{1-\gamma}\Big\{1-\exp\Big[-C_2\Big(\frac{k}{S^2\zeta^2}\Big)^{C_3}\Big]\Big\}\frac{\Omega\zeta^2}{v}\rho\sqrt{(1-\gamma)k}\zeta S. \tag{2.3}$$

In the full turbulence region, the streaky fluctuations will break down and the kinetic energy of non-turbulent fluctuation must be zero, so a dissipation term for the non-turbulent fluctuation transport equation is introduced to simulate the break down procession. For the channel turbulence flow, Chien [34] showed that the expression for the viscous dissipation of turbulent kinetic energy is as follows:

$$\varepsilon=2\nu\frac{k_t}{y^2}.$$

Based on the dimension analysis, the suitable dissipation term for Transport equation of the non-turbulent viscosity might be

$$\varepsilon_{\nu_L}=C_4 f_{V_L}\rho\nu_L\frac{\sqrt{k_L}}{y}=C_4 f_{V_L}\rho\nu_L\frac{\sqrt{\gamma k}}{y}, \tag{2.4}$$

where $f_{\nu_L}=C_5(\nu_L/\nu)^4(Sy^2/\nu)^2+C_6(\nu_t/\nu)^3$. The term is seen as non-active in the laminar region and small in the fully turbulent region.

2.2 Transport equation for the intermittency factor γ

A transport equation for the intermittency factor γ is developed to simulate the bypass transition process. The transport equation for the intermittency factor is given here as

$$\frac{\partial(\rho\gamma)}{\partial t}+\frac{\partial(\rho u_j\gamma)}{\partial x_j}=D_\gamma+P_\gamma-\varepsilon_\gamma, \tag{2.5}$$

where D_γ, P_γ and ε_γ are the diffusion, production and dissipation terms, respectively.

Based on the experimental data, the streamwise development of γ in the transition region appears to be quite universal in boundary layer flows for quite a large range of flows, e.g., with favorable and adverse pressure gradients, and can be described with the following well-known empirical correlation of Dhawan & Narasimha [15]:

$$\gamma=1-\exp\Big[-\frac{(x-x_{tr})^2 n\sigma}{U_e}\Big],$$

where U_e is the velocity at the boundary layer edge, n is the turbulent spot formation rate, the subscript t stands for the transition onset, and σ represents the spot propagation parameter. The streamwise derivation of equation above is as follows:

$$\frac{d\gamma}{dx}=2\sqrt{\frac{n\sigma}{U_e}}(1-\gamma)[-\ln(1-\gamma)]^{0.5}.$$

Based on the empirical correlation of Mayle [25], $\hat{n}\sigma$ represents the effect from free stream turbulence. Production term and dissipation term in intermittency transport equation are defined as:

$$P_\gamma=C_7 f(Tu_\infty)[-\ln(1-\gamma)]^{0.5}F_{onset}\Big(\frac{\Omega\zeta^2}{v}\Big)S, \quad \varepsilon_\gamma=\gamma P_\gamma, \tag{2.6}$$

where the dumping function F_{onset} is used to determine the transition onset:

$$F_{onset}=1.0-\exp\Big[-C_8\Big(\frac{\sqrt{k}\zeta}{\nu}\Big)\Big(\frac{|\nabla k|}{S|U|}\Big)\Big].$$

The effect of free stream turbulence is taken into account for the development of intermittency factor, $f(Tu_\infty)$ represents the spot-formation rate and is given by Mayle [25] as

$$f(Tu_\infty)=\sqrt{1.25\times 10^{-11}\times(Tu_\infty)^{7/4}}.$$

The diffusion term is designed as the standard gradient-type form:

$$D_\gamma=\frac{\partial}{\partial x_i}\Big[\Big(\mu+\sigma_{\gamma 1}\mu_L+\sigma_{\gamma 2}\mu_T\Big)\frac{\partial\gamma}{\partial x_i}\Big], \quad \sigma_{\gamma 1}=\sigma_{\gamma 2}=1.0. \tag{2.7}$$

Table 1 shows the transition model constants.

Table 1: Model constants.

σ_{L1}	σ_{L2}	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
1.0	1.0	8.0e-5	29.375	4.0	0.01	0.0055	0.01	56.0	1.8

2.3 Reynolds-averaging Navier-Stokes and turbulence model equations

The single-phase flow governed by the steady Reynolds-averaged continuity and momentum equations is solved here. The linear eddy-viscosity concept is applied to model the Reynolds stresses. The governing equations are shown below. The equation for the conservation of mass:

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho\bar{U}_j)}{\partial x_j}=0. \tag{2.8}$$

The equation for the conservation of momentum:

$$\frac{\partial(\rho\bar{U}_i)}{\partial t}+\frac{\partial(\rho\bar{U}_j\bar{U}_i)}{\partial x_j}=-\frac{\partial P}{\partial x_i}+\frac{\partial}{\partial x_j}\Big[(\mu+\mu_{eff})\Big(\frac{\partial\bar{U}_i}{\partial x_j}+\frac{\partial\bar{U}_j}{\partial x_i}\Big)-\frac{2}{3}(\mu+\mu_{eff})\frac{\partial\bar{U}_l}{\partial x_l}\delta_{ij}\Big]. \tag{2.9}$$

The effective viscosity μ_{eff} is the sum of the non-turbulent and the turbulent viscosities:

$$\mu_{eff}=\mu_L+\mu_t. \tag{2.10}$$

It is noted here that the present approach converts to the SST model when the flow becomes fully turbulent. The last model transport equation is thus the SST turbulence model [35] which is used to compute the turbulent viscosity in this bypass transition model package. To account for the transition effect, the SST turbulence model is modified as:

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho u_j k)}{\partial x_j}==\frac{\partial}{\partial x_j}\Big\{[\mu+\sigma_k(\mu_L+\mu_T)]\frac{\partial\gamma}{\partial x_j}\Big\}+P_{nt}+P_k-\varepsilon, \tag{2.11a}$$

$$\frac{\partial(\rho\omega)}{\partial t}+\frac{\partial(\rho u_j\omega)}{\partial x_j}=\frac{\partial}{\partial x_j}\Big[(\mu+\sigma_\omega\mu_T)\frac{\partial\omega}{\partial x_j}\Big]+P_\omega-D_\omega+Cd_\omega, \tag{2.11b}$$

$$\mu_t=\frac{a_1\rho(\gamma k)}{\max(a_1\omega,SF_2)}, \tag{2.11c}$$

$$P_{nt}=2\mu_L S_{ij}S_{ij}, \tag{2.11d}$$

where P_{nt} represents the production term of the laminar kinetic energy in the pre-transitional boundary layer region and diminishes when the flow goes turbulent.

2.4 Summary of the four-equation bypass turbulence transition model

The four equation bypass turbulence transition model includes the transport equations for the non-turbulent viscosity, the intermittency factor γ, and the fluctuating kinetic energy k, the specific dissipation rate ω

$$\frac{\partial(\rho\nu_L)}{\partial t}+\frac{\partial(\rho U_i\nu_L)}{\partial x_i}=P_{bypass}-\varepsilon_{\nu_L}+\frac{\partial}{\partial x_i}\Big[\rho(\nu+\sigma_{L1}\nu_L+\sigma_{L2}\nu_T)\frac{\partial\nu_L}{\partial x_i}\Big], \tag{2.12a}$$

$$\frac{\partial(\rho\gamma)}{\partial t}+\frac{\partial(\rho u_j\gamma)}{\partial x_j}=D_\gamma+P_\gamma-\varepsilon_\gamma, \tag{2.12b}$$

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho u_j k)}{\partial x_j}==\frac{\partial}{\partial x_j}\Big\{[\mu+\sigma_k(\mu_L+\mu_T)]\frac{\partial\gamma}{\partial x_j}\Big\}+P_{nt}+P_k-\varepsilon, \tag{2.12c}$$

$$\frac{\partial(\rho\omega)}{\partial t}+\frac{\partial(\rho u_j\omega)}{\partial x_j}=\frac{\partial}{\partial x_j}\Big[(\mu+\sigma_\omega\mu_T)\frac{\partial\omega}{\partial x_j}\Big]+P_\omega-D_\omega+Cd_\omega, \tag{2.12d}$$

with

$$\mu_{eff}=\mu_L+\mu_t, \qquad P_{nt}=2\mu_L S_{ij}S_{ij},$$
$$\mu_L=\rho\nu_L, \qquad \mu_t=\frac{a_1\rho(\gamma k)}{\max(a_1\omega,SF_2)}.$$

The effective viscosity $\mu_{eff}=\mu_L+\mu_t$ is an important variable to predict the turbulence transition. In the pre-transitional region, $\gamma\approx 0$, $\mu_t\approx 0$, and the non-turbulent viscosity μ_L can be determined by the transport equation (2.1), so we can deduce that $\mu_{eff}\approx\mu_L$, $k\approx k_L$. Then in the transitional region, the intermittency factor γ will be triggered by the developments of the k and the mean flow, then the turbulent fluctuation energy will be amplified by the intermittency factor γ. And in the fully turbulent region ($\mu_L\approx 0$, $\gamma\approx 1.0$), the present transition model will be converted to the standard SST model.

3 Test cases

The ERCOFTAC test cases of T3AM, T3A and T3B [36], classified as the low, moderate and high free stream turbulence intensity cases, have been widely employed to validate the performance of the bypass transition models. These cases are also considered in the present work. All cases are boundary layer flows on a flat plate under zero-pressure gradient condition. Fig. 1 shows the details of the grid configuration and boundary conditions. The inlet conditions of all the three cases are specified in Table 2.

The numerical approach adopted in this work is similar to Fu & Wang [23]. AUSM+ and central difference scheme are used to discretize the convection and diffusion terms in the governing equations. In all cases, the first node adjacent to the wall is located at $y+$ below 0.3. The velocity at inlet and static pressure at outlet are specified constants. New boundary conditions should be specified for the intermittency factor and the non-turbulent viscosity. The value of the intermittency factor and non-turbulent at inlet are given as 0.001 (or another small value) and zero respectively, and the zero-normal-gradient condition is applied for the intermittency factor and the non-turbulent at the

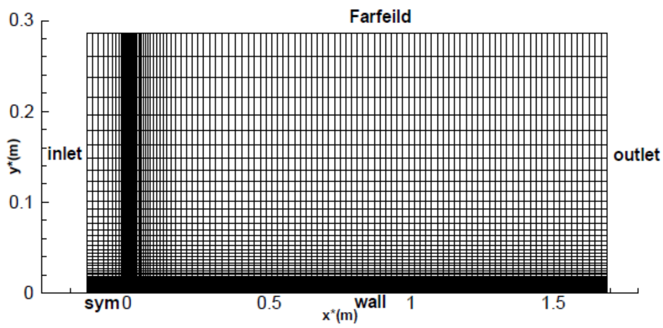

Figure 1: Grid configuration and boundary conditions.

Table 2: Inlet condition for the flat-plate boundary layer with zero pressure gradient.

Case	U_{in} (m/s)	FST(%)	Density (kg/m^3)	Dynamic viscosity (kg/ms)	μ_t/μ
T3AM	19.8	0.874	1.2	1.8e-5	8.72
T3A	5.4	3.5	1.2	1.8e-5	12.0
T3B	9.4	6.5	1.2	1.8e-5	100.0

solid boundary. Fig. 2 shows the turbulence decay in the free-stream outside the boundary layer using the present turbulence transition model, and the results show good agreement with the experiment results.

The present results will be compared with the results from the well-known transition model based on local variables, Menter et al.'s model [27] and Walters-Leylek model [29]. Figs. 3, 4 and 5 show the velocity profiles at different streamwise locations. The present model gives good agreement with the experiments for all three cases in the pre-transitional region. In the transition and post-transition regions, velocity profiles agree well with the experimental data near the wall but are slightly over-predicted in the boundary layer region. At the boundary layer edge, the streamwise velocities become over-saturated in the post-transition regions because too much non-turbulent viscosity is produced by the model. The new transition model also gives satisfactory results for the boundary layer development along the streamwise direction as shown by the momentum thickness Reynolds numbers (Fig. 6), and Walters-Leylek model [29] did not provide the data of the momentum thickness Reynolds numbers. We can observe that the transition starts at the location where the moment thickness Reynolds number line deviates from the laminar line. Based on the experimental data, the transition Reynolds numbers are 1.4e6, 1.35e5 and 5.9e5 corresponding to the momentum thickness Reynolds numbers of 810, 272, and 180 for T3AM, T3A and T3B cases, respectively. For the T3AM case, note that the transition process is not yet complete within the flat plate boundary layer flow as the length of the flat plate is 170cm in the experiment. Figs. 5 and 6 show that the moment thickness Reynolds number provided by the present model is always small than the experimental result in the post-transitional region, because the streamwise velocity profiles are more saturated near the boundary layer edge (in Figs. 3, 4 and 5).

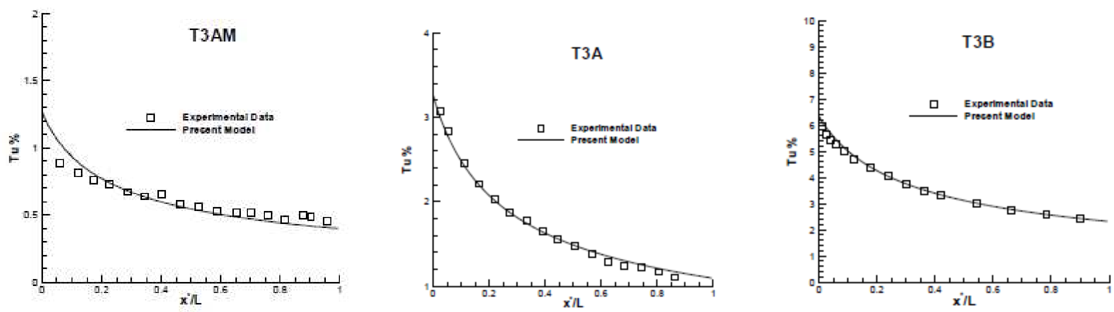

Figure 2: Streamwise decay of freestream turbulence for T3AM, T3A, T3B.

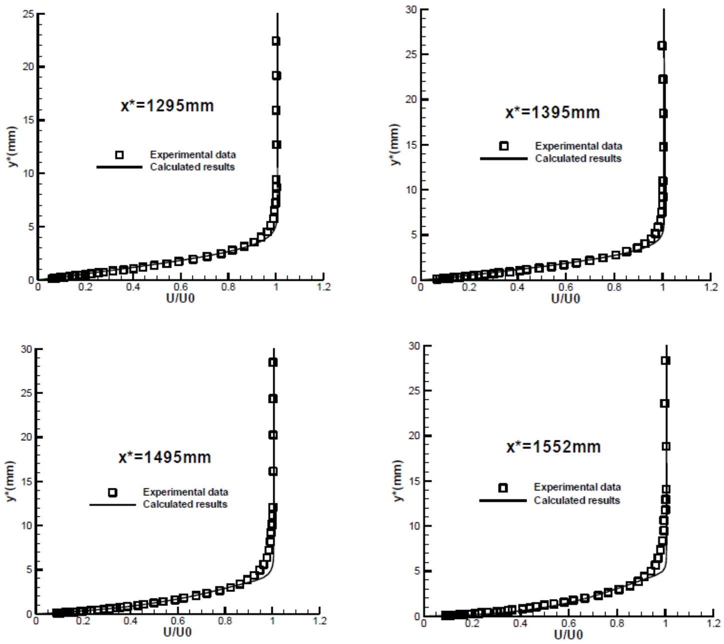

Figure 3: Non-dimensional streamwise velocity profiles for T3AM. $x* = 1295$mm: pre-transition region; $x* =$ 1395mm, 1495mm, 1552mm: transition region.

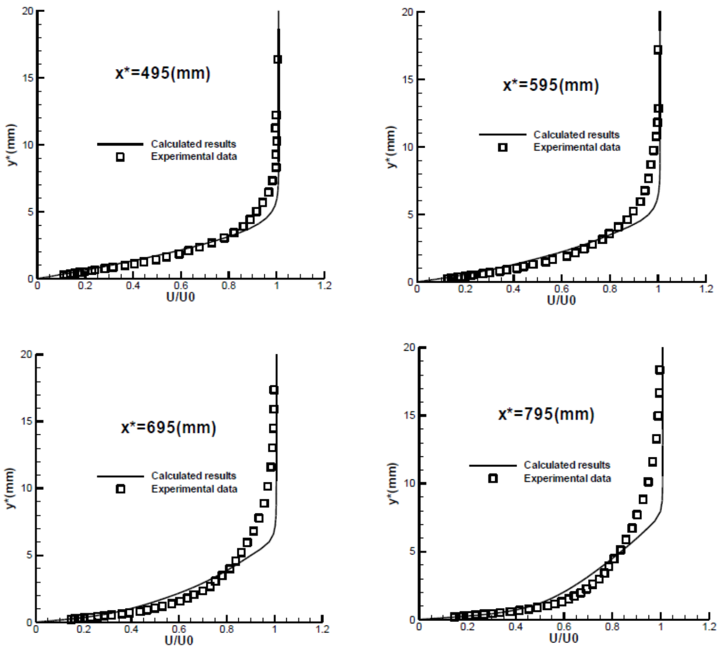

Figure 4: Non-dimensional streamwise velocity profiles for T3A. $x* = 495$mm, 595mm: transition region; $x* =$ 695mm, 795mm: post-transition region.

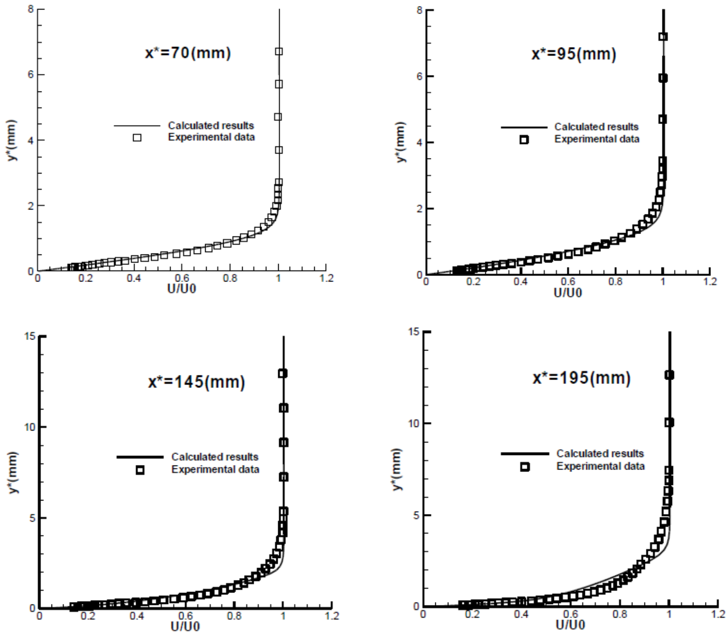

Figure 5: Non-dimensional streamwise velocity profiles for T3B. $x*=70$mm, 95mm: pre-transition region; $x*=145$mm: transition region; $x*=195$mm: post-transition region.

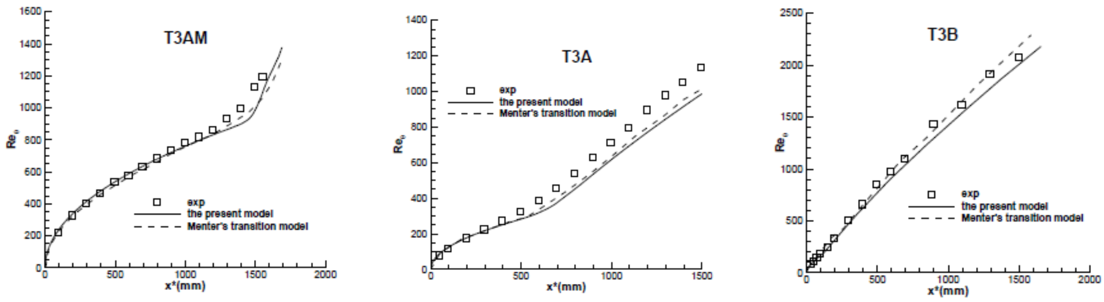

Figure 6: Momentum thickness Reynolds number for T3AM, T3A and T3B.

The transition onset is often regarded as the best indication for the success or failure in the transition prediction. Fig. 7 shows the distribution of the skin frictions for all the cases and the comparisons between the experimental data, Menter et al.'s Model, Walter-Leylek's Model and the present model. Menter et al.'s Model almost gave good transition prediction for all the cases apart from the T3B case for the high level of incoming turbulence, and Walter-Leylek Model just gave good transition prediction for the T3B case for the high level of incoming turbulence. The model of Menter was constructed based on the empirical correlation and does not consider the physical processes of the bypass transition, it always provides the early transition onset location for the high free stream turbulence intensity. The present transition model in this paper gives a little delayed

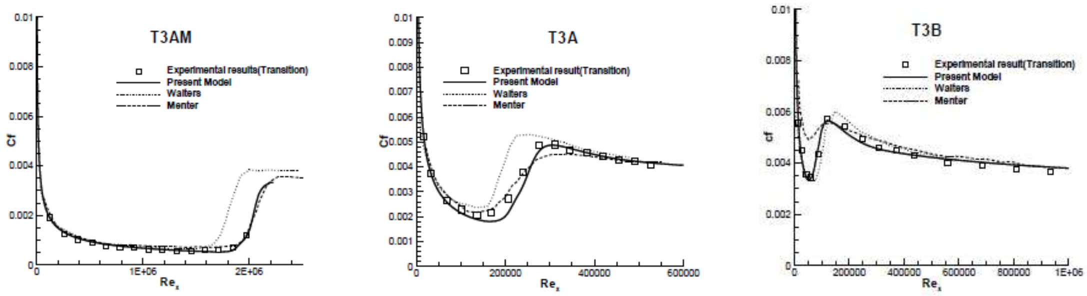

Figure 7: Distribution of skin friction coefficient for T3AM, T3A and T3B.

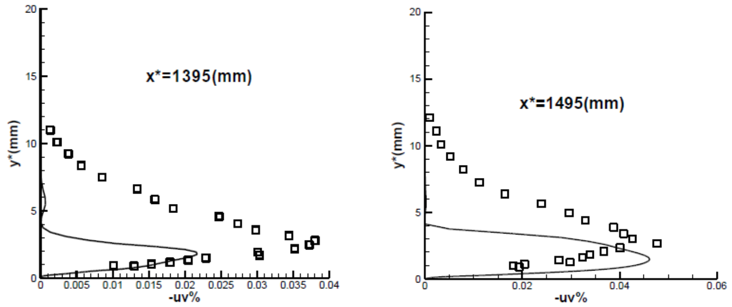

Figure 8: Streamwise profile of non-dimensional shear stress for T3AM. $x* = 1395$mm, 1495mm: transition region.

onset of transition in the cases of low and moderate free stream turbulence intensity, so the Reynolds shear stress provided by the present model is somehow under-predicted in the transition regions (Figs. 8, 9 and 10). For the Reynolds shear stress in the post-transitional region, the over-predicted results are given by the present model, because the present model is converted to the SST model in the fully turbulent region and the original SST model gives over-predicted result for all cases.

Fig. 11 represents the computation results of intermittency factor γ, the intermittency factor γ is near zero in the pre-transition region and the intermittency factor $\gamma = 1$ in the fully turbulence region. Fig. 12 represents the computation results of non-turbulent viscosity, we can observe that the non-turbulent viscosity is near zero in the laminar region and fully turbulent region as the streaky fluctuations will break down and will not exist in the fully turbulent region, so we can deduce that the transport equation of non-turbulent viscosity can predict the correct distribution and is responsible for the bypass transition process. Due to the dissipation term of the non-turbulent viscosity transport equation, the value of the non-turbulent viscosity will be zero in the post-transitional region what is physical and different to Warren & Hassan [29] and Fu & Wang [30] models. But we can also observe that the non-turbulent viscosity does not decay quickly outside the boundary layer in the fully turbulent region in Fig. 12, and the dissipation term of the non-turbulent viscosity transport equation should be improved in the future.

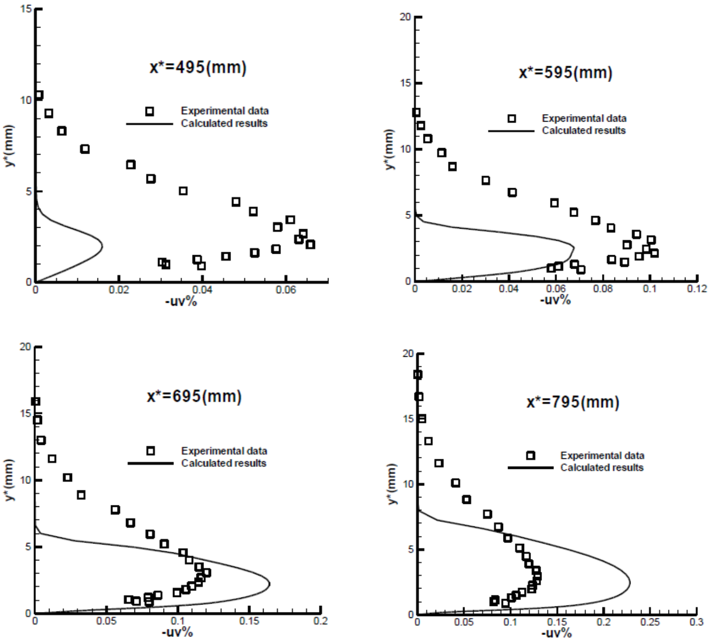

Figure 9: Streamwise profiles of non-dimensional shear stress for T3A. $x*$=495mm, 595mm: transition region; $x*$=695mm, 795mm: post-transition region.

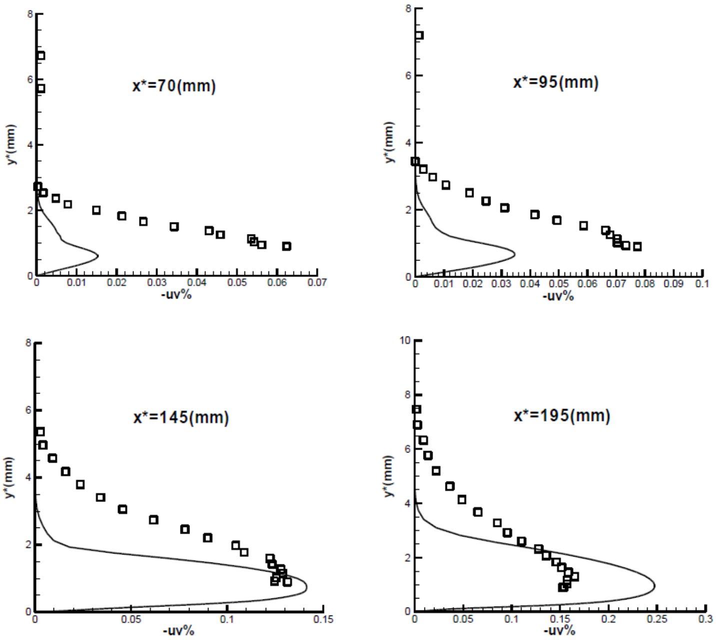

Figure 10: Streamwise profiles of the non-dimensional shear stress for T3B. $x*=70$mm, 95mm: pre-transition region; $x*=145$mm: transition region; $x*=195$mm: post-transition region.

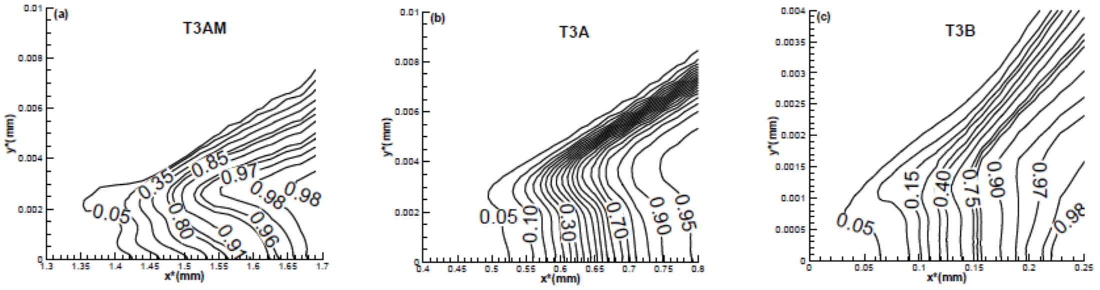

Figure 11: Contour of intermittency for T3AM, T3A, T3B.

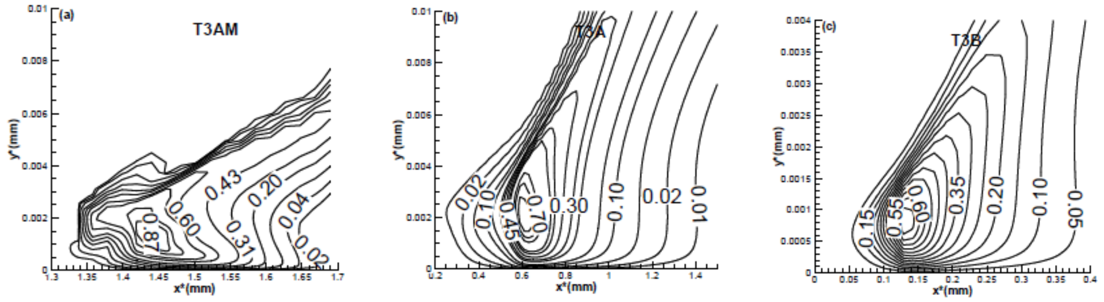

Figure 12: Contour of non-turbulent viscosity for T3AM, T3A, T3B.

4 Summary and conclusions

At higher freestream turbulence levels, only bypass transition is pertinent in boundary layer flow. And the low-frequency non-turbulent fluctuation exists in the region of pre-transitional boundary layer flow which has been found in lots of experimental and numerical results. Mayle & Schulz [28] and Walters & Leylek [29] proposed a transport equation for the kinetic energy of low-frequency non-turbulent fluctuation to predict the bypass transition. Here, a new locally formulated transport equation for the non-turbulent viscosity of the low-frequency non-turbulent fluctuation, rather than the kinetic energy equation for bypass transition, is developed to describe the development of the streak and predict the bypass transition in boundary layer flow. Based on local variables, the new transition model can be easily implemented in the current CFD code. It is shown that the present transition model gives better results over a range of incoming freestream turbulence intensities.

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