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The effect of temperatures and magnetic field on relativistic Fermi gas

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Abstract. By using the methods of theoretical analysis and numerical simulation, the relativistic effects of the thermodynamic properties of Fermi gas in a strong magnetic field are studied, and the influential mechanisms of temperatures and the magnetic field on the relativistic effects of the system are analyzed. It is shown that, at low temperatures, the effects of temperatures on the relativistic effect relate to the chemical potential of the free system. The magnetic field's effects on the relativistic effects of heat capacity and chemical potential display oscillating characteristics. At high temperatures, the relativistic effects of the thermodynamic quantities trigger oscillations with increasing of temperatures and magnetic field, and its amplitude decreases obviously to disappearing with the increasing of temperatures. However, its amplitude increases evidently due to the amplified magnetic field.

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Key words: Fermi gas, relativity, magnetic field, temperature

1 Introduction

Recently, researches on ultra-cold Fermi gases have obtained abundant achievements [1-8], and the studies of relativity Fermi gas have been a hot field due to the discovery of neutrino. Under the condition of considering the relativistic effect, we have investigated the thermodynamic properties and stability of a weakly interacting Fermi gas in a weak magnetic field [9,10] researched weakly interacting Fermi gas's relativistic paramagnetism [11], and studied relativistic thermodynamic properties of Fermi gas in hard -sphere potential by using the methods of quantum statistics, thermodynamic theories and numerical simulation, and also analyzed the effects of temperature, as well as the

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relativistic influence on the thermodynamic properties of the system [12]. We also researched relativistic thermodynamic properties of Fermi gas in a strong magnetic field at low temperatures, gave the corresponding analytical expression, and analyzed the influences of relativistic effect on the thermodynamic properties [13]. On this basis, using the methods of theoretical analysis and numerical simulation, we studied relativistic effect of the thermodynamic properties of Fermi gas in a strong magnetic field at both high and low temperatures, and then explored the influential mechanism of temperatures and the magnetic field on the relativistic effect.

2 The analytical expressions of the thermodynamic properties

According to Ref. [13], the relativistic part of the thermodynamic potential function of system can be shown as following

$$\begin{aligned}\Omega^R(\mu)=&\frac{(\sigma B)^{5/2}}{8m^2c^2}\left(\frac{2m}{\pi}\right)^{5/2}\Gamma\left(\frac{5}{2}\right)\frac{TV}{\pi^2\hbar^3}\left\{-\frac{1}{\sqrt{2}}\log[1+e^{\frac{\mu}{T}}]\sum_{k=1}^{\infty}\frac{1}{k^{5/2}}\right.\\&\left.+\frac{\sigma B}{\sqrt{2}\pi T}\frac{e^{\mu/T}}{1+e^{\mu/T}}\sum_{k=1}^{\infty}\frac{1}{k^{7/2}}-\sum_{k=1}^{\infty}\frac{\pi}{k^{5/2}}\frac{\sin(\pi k\mu/\sigma B-5\pi/4)}{sh(\pi^2kT/\sigma B)}\right\},\end{aligned}\tag{1}$$

and at low temperatures, the items of relativistic effect of energy, heat capacity, chemical potential can be expressed respectively as

$$\begin{aligned}U_1^R=-T^2\frac{\partial}{\partial T}\left[\frac{\Omega^R}{T}\right]_{V,N}=&-\frac{3(\sigma B)^{5/2}\sqrt{m}}{8\pi^4\hbar^3c^2}V\left[\mu\sum_{k=1}^{\infty}\frac{1}{k^{5/2}}-\frac{\sigma B}{\pi}\sum_{k=1}^{\infty}\frac{1}{k^{7/2}}\right.\\&\left.+\frac{\sqrt{2}\pi^2T^2}{\sigma B}\sum_{k=1}^{\infty}\frac{\sin\alpha\cos h\beta}{k^{3/2}\sin h^2\beta}\right],\end{aligned}\tag{2}$$

$$\begin{aligned}C_1^R=\left[\frac{\partial U}{\partial T}\right]_{V,N}=&-\frac{3(\sigma B)^{3/2}\sqrt{2m}}{4\pi\hbar^3c^2}TV\left[\sum_{k=1}^{\infty}\frac{\sin\alpha\cos h\beta}{k^{3/2}\sin h^2\beta}\right.\\&\left.+\frac{\pi^2T}{2\sigma B}\sum_{k=1}^{\infty}\frac{\sin\alpha(\sin h^2\beta-2\cos h^2\beta)}{k^{1/2}\sin h^3\beta}\right]\end{aligned}\tag{3}$$

$$u_1^R=[\frac{\partial\Omega^R}{\partial N}]_{V,T}=-\frac{3(\sigma B)^{5/2}\sqrt{m}\varepsilon_F}{4\pi^4\hbar^3c^2n}V\left[\sum_{k=1}^{\infty}\frac{1}{k^{5/2}}+\frac{\sqrt{2}\pi^2T}{\sigma B}\sum_{k=1}^{\infty}\frac{\cos\alpha}{k^{3/2}\sin h\beta}\right]\tag{4}$$

where $\alpha=\pi\mu k/\sigma B-\pi/4$, $\beta=\pi^2kT/\sigma B$.

Choosing $\mu=x\sigma B$, $T=y\sigma B$, then the expressions (2),(3)and (4) are written as

$$U_1^R=-\frac{3(\sigma B)^{7/2}\sqrt{m}}{8\pi^4\hbar^3c^2}V\left[x\sum_{k=1}^{\infty}\frac{1}{k^{5/2}}-\frac{1}{\pi}\sum_{k=1}^{\infty}\frac{1}{k^{7/2}}+\frac{\sqrt{2}\pi^3y^2}{\sigma B}\sum_{k=1}^{\infty}\frac{\sin(\pi kx-\pi/4)\cos h(\pi^2ky)}{k^{3/2}\sin h^2(\pi^2ky)}\right],\tag{5}$$

$$C_1^R = -\frac{3(\sigma B)^{5/2}\sqrt{2m}}{4\pi\hbar^3 c^2} V \Bigg\{ y \sum_{k=1}^{\infty} \frac{\sin(\pi kx - \pi/4)\cos h(\pi^2 ky)}{k^{3/2}\sin h^2(\pi^2 ky)} + \frac{\pi^2 y^2}{2} \sum_{k=1}^{\infty} \frac{\sin(\pi kx-\pi/4)[\sin h^2(\pi^2 ky) - 2\cos h^2(\pi^2 ky)]}{k^{1/2}\sin h^2(\pi^2 ky)} \Bigg\}, \quad (6)$$

$$u_1^R = -\frac{3(\sigma B)^{5/2}\sqrt{m}\varepsilon_F}{4\pi^4\hbar^3 c^2 n} V \left[\sum_{k=1}^{\infty} \frac{1}{k^{5/2}} + \sqrt{2}\pi^3 y \sum_{k=1}^{\infty} \frac{\cos(\pi kx - \pi/4)}{k^{3/2}\sin h^2(\pi^2 ky)} \right]. \quad (7)$$

Choosing $T = 10^{-2}T_F \approx 10^{-2}\mu,\ \mu z = \sigma B$, the expressions (2),(3) and (4) can also be expressed as

$$U_1^R = -\frac{3(\mu)^{7/2}\sqrt{m}}{8\pi^4\hbar^3 c^2} V \left[z^{5/2} \sum_{k=1}^{\infty} \frac{1}{k^{5/2}} - \frac{z^{7/2}}{\pi} \sum_{k=1}^{\infty} \frac{1}{k^{7/2}} + \frac{\sqrt{2}\pi^2 z^{3/2}}{10^4} \sum_{k=1}^{\infty} \frac{\sin(\pi k/z - \pi/4)\cos h(10^{-2}\pi^2 k/z)}{k^{3/2}\sin h^2(10^{-2}\pi^2 k/z)} \right], \quad (8)$$

$$C_1^R = -10^{-2} \frac{3\mu^{5/2}\sqrt{2m}}{4\pi\hbar^3 c^2} V \Bigg\{ z^{3/2} \sum_{k=1}^{\infty} \frac{\sin(\pi k/z - \pi/4)\cos h(10^{-2}\pi^2 k/z)}{k^{3/2}\sin h^2(10^{-2}\pi^2 k/z)} + \frac{\pi^2 z^{1/2}}{200} \sum_{k=1}^{\infty} \frac{\sin(\pi kz - \pi/4)[\sin h^2(10^{-2}\pi^2 k/z) - 2\cos h^2(10^{-2}\pi^2 k/z)]}{k^{1/2}\sin h^3(10^{-2}\pi^2 k/z)} \Bigg\}, \quad (9)$$

$$u_1^R = -\frac{3(\mu)^{7/2}\sqrt{m}}{4\pi^4\hbar^3 c^2 n} \left[z^{5/2} \sum_{k=1}^{\infty} \frac{1}{k^{5/2}} + \frac{\sqrt{2}\pi^2 z^{3/2}}{100} \sum_{k=1}^{\infty} \frac{\cos(\pi k/z - \pi/4)}{k^{3/2}\sin h(\pi^2 k/20)} \right]. \quad (10)$$

At high temperatures, $e^{\mu/T} \ll 1$ the expression (1) is written as

$$\Omega^R(\mu) = -\frac{3(\sigma B)^{5/2} m^{1/2}}{8\pi^4\hbar^3 c^2} TV \left[e^{\mu/T}\varsigma(\frac{5}{2}) - \frac{\sigma B}{\pi T} e^{\mu/T}\varsigma(\frac{7}{2}) - \sqrt{2}\sum_{k=1}^{\infty} \frac{\pi}{k^{5/2}} \frac{\sin\alpha}{\sin h\beta} \right], \quad (11)$$

supposing Boltzmann constant $K=1$, there are $\mu = T\ln(n\lambda^3/2)$. Consequently, we get

$$U_2^R \approx \frac{3(\sigma B)^{5/2} m^{1/2}}{8\pi^4\hbar^3 c^2} V \left[-\frac{3}{2} Te^{\mu/T} \sum_{k=1}^{\infty} \frac{1}{k^{5/2}} + \frac{5\sigma B}{2\pi} e^{\mu/T} \sum_{k=1}^{\infty} \frac{1}{k^{7/2}} - \frac{\sqrt{2}}{\sigma B} \mu T \sum_{k=1}^{\infty} \frac{\pi^2}{k^{3/2}} \frac{\cos\alpha}{\sin h\beta} \right], \quad (12)$$

$$C_2^R \approx \frac{3(\sigma B)^{5/2} m^{1/2}}{8\pi^4\hbar^3 c^2} V \Bigg\{ \frac{3}{4} e^{\mu/T} \left[\sum_{k=1}^{\infty} \frac{1}{k^{5/2}} - 5\frac{\sigma B}{\pi T} \sum_{k=1}^{\infty} \frac{1}{k^{7/2}} \right] - \frac{2\sqrt{2}\mu}{\sigma B} \sum_{k=1}^{\infty} \frac{\pi^2}{k^{3/2}} \frac{\cos\alpha}{\sinh\beta} + \frac{\sqrt{2}\mu^2}{(\sigma B)^2} \left[\sum_{k=1}^{\infty} \frac{\pi^3}{k^{1/2}} \frac{\sin\alpha}{\sinh\beta} + \frac{T}{\mu} \sum_{k=1}^{\infty} \frac{\pi^4}{k^{1/2}} \frac{\cos\alpha\cos h\beta)}{\sin h^2\beta} \right] \Bigg\} \quad (13)$$

$$u_2^R \approx -\frac{3(\sigma B)^{5/2} m^{1/2}}{8\pi^4\hbar^3 c^2} \frac{T}{n} \{ e^{\mu/T} \left[\sum_{k=1}^{\infty} \frac{1}{k^{5/2}} - \frac{\sigma B}{\pi T} \sum_{k=1}^{\infty} \frac{1}{k^{7/2}} \right] - \sqrt{\frac{T}{\sigma B}} \sum_{k=1}^{\infty} \frac{\pi^2}{k^{3/2}} \frac{\cos\alpha}{\sinh\beta}. \quad (14)$$

3 Numerical simulation, analysis and discussion

3.1 Figs at low temperatures

Supposing $\sigma B = 10^{-2}\mu = 10^{-2}\varepsilon_F \sim 10^{-29}$, x=103,111,117; x=106,110,114; y:0∼1, according to the expressions (5)-(7), we draw figs of thermodynamic quantities-temperatures.

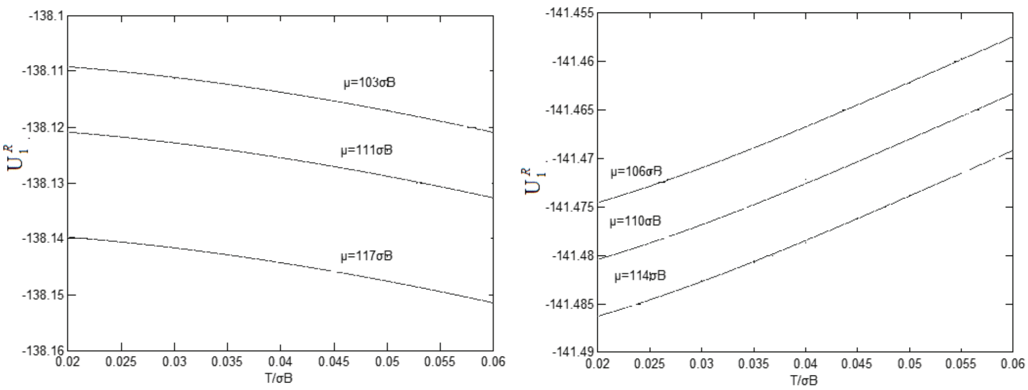


Figure 1: The curves of total energy U_1^R versus temperature.

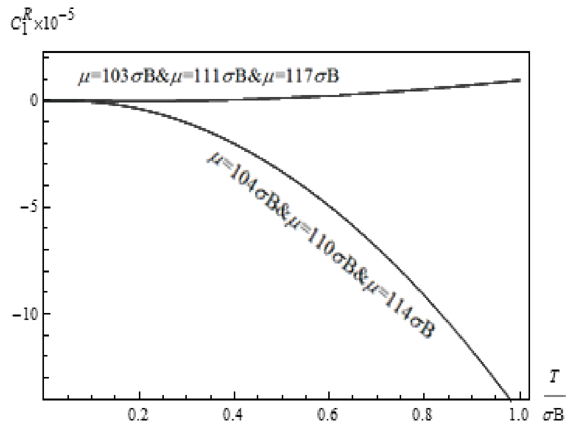


Figure 2: The curves of heat capacity C_1^R versus temperature.

Supposing $\mu \sim \varepsilon_F \sim 10^{-27}$, and $z:(1\sim 9)10^{-2}$, based on the expressions (8)-(10), we draw figs of thermodynamic quantities - magnetic field.

3.2 Figs at high temperatures

According to the expressions (12)-(14), and supposing $n = 10^{15}$, we draw figs of at high temperatures ($T \gg T_F$). In addition, thermodynamic qualities versus the temperature are completed with the condition of B equating to $1T$; thermodynamic qualities versus the

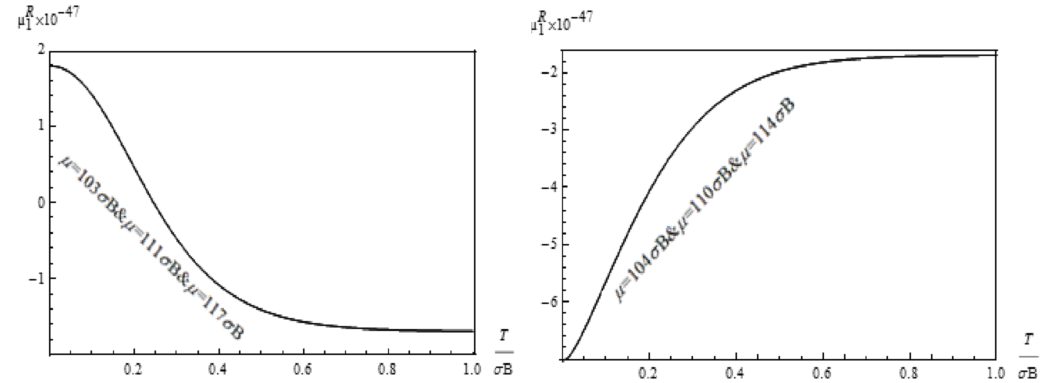


Figure 3: The curves of chemical potential μ_1^R versus temperature.

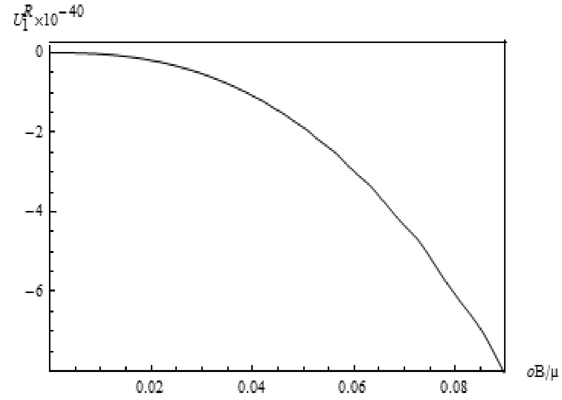


Figure 4: The oscillation of total energy U_1^R versus magnetic field.

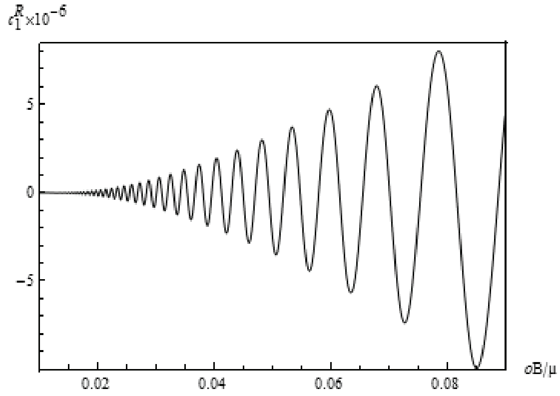


Figure 5: The oscillation of total energy C_1^R versus magnetic field.

magnetic field are given when $T=1K^0$.

3.3 Analysis the influences of temperatures and magnetic field on the relativistic effect at low temperatures

Figs. 1-3 show, under the condition of $\mu \gg \sigma B$, the characteristic of the relativistic effect of the thermodynamic properties with the change of temperature relates to the chemical potential of the free system and further related to the density of particle number. For the energy, when the chemical potential of the free system is odd/even (i.e. the chemical potential of the free system is odd/even times of σB), the absolute value of the relativistic effect increases/decreases along with temperature increasing. In addition, the greater the chemical potential of the free system is, the greater the absolute value of the relativistic effect will be. For the heat capacity, when the free system's chemical potential is odd, the relativistic effect is positive with increasing temperature; however, it has negative

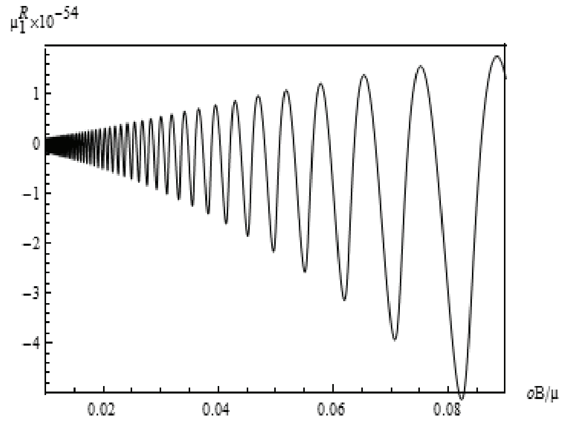
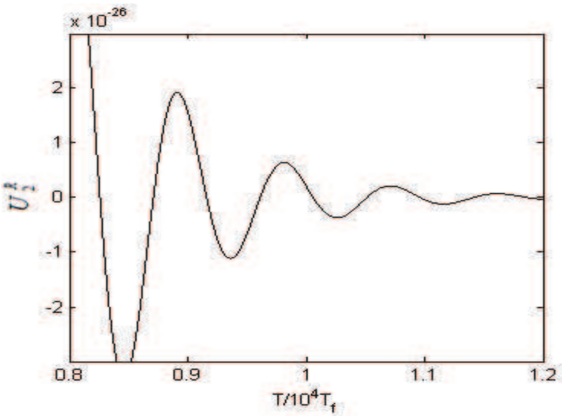


Figure 6: The oscillation of chemical potential μ_1^R versus magnetic field



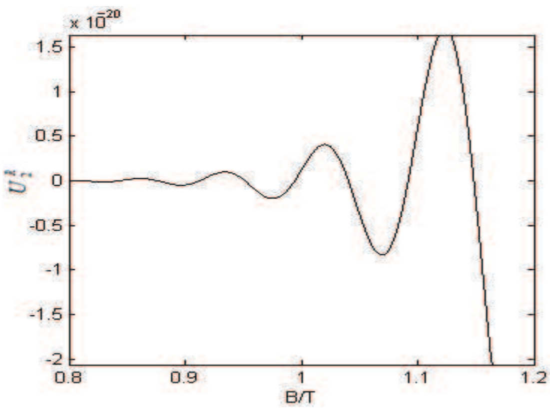


Figure 7: The curves of energy U_2^R versus temperature. Figure 8: The curves of energy U_2^R versus magnetic field.

effect when the free system's chemical potential is even. When it comes to the system's chemical potential, if it is odd, with increasing temperature, the relativistic effect above turn to negative from positive; if even, with the increasing temperature, the absolute value of becomes smaller and the relativistic effect is negative. Figs 4, 5 and 6 reveal that, in low temperatures, the relativistic effects on heat capacity and chemical potential is stronger and its oscillation also becomes more obviously with increasing of the magnetic field; those effects on energy are negative and stronger due to increasing of the magnetic field, however, does not show oscillating characteristic.

Figs. 4-6 indicate, due to the magnetic field varying, the relativistic effects of the thermodynamic properties present oscillatory characteristic. For energy, the relativistic effects contain both oscillatory and no-oscillatory part. Total effects are the absolute value of the relativistic effect increasing in a wave-like manner which has trend of increasing

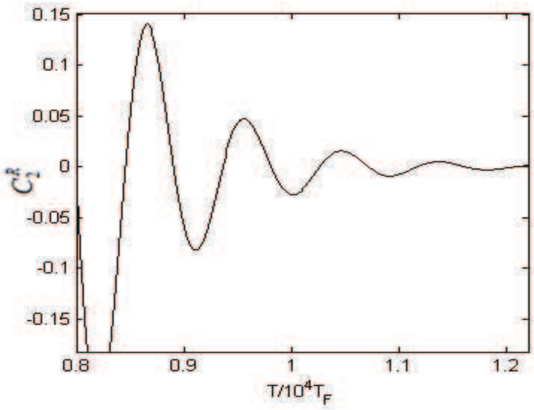


Figure 9: The curves of heat capacity C_2^R versus temperature.

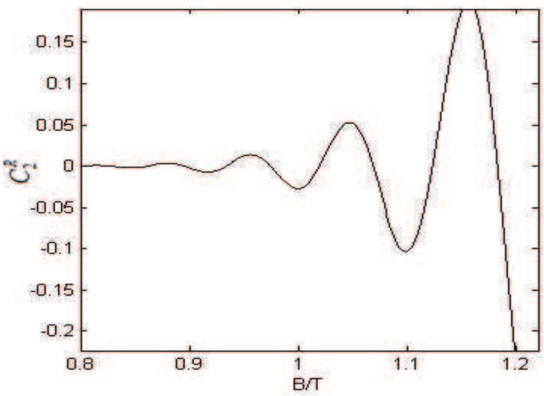


Figure 10: The curves of heat capacity C_2^R versus magnetic field.

with the strengthening of the magnetic field, because the oscillatory part is weaker than no-oscillatory part. For the heat capacity, the relativistic effect has obvious oscillation along with strengthening magnetic field, and the oscillatory amplitude has the trend of becoming bigger. As for the chemical potential of the system, oscillations of the relativistic effect due to strengthening magnetic field are not regular. Firstly, the oscillatory center continual moves down. Secondly, the oscillatory amplitude continually increases. Thirdly, the oscillatory top is smooth. On the contrary, the oscillatory bottom is keen.

3.4 Analysis of the influences of temperature and magnetic field on the relativistic effect at high temperatures

By Figs. 7-12, at high temperatures, it is known that, the relativistic effect of energy, heat capacity and chemical potential with the increase of temperature and magnetic field trigger oscillations which are very obvious. The amplitude of the relativistic effect decreases evidently with the increasing temperature, and finally disappears. In contrast, the amplitude of the relativistic effect increases evidently with strengthening magnetic field.

4 Conclusions

By using the methods of theoretical analysis and numerical simulation, we have studied the relativistic effect of the thermodynamic properties of Fermi gas in a strong magnetic field, analyzed the influential mechanism of temperatures and the magnetic field on the relativistic effect. It is shown that, at low temperatures, the influences of temperatures on the relativistic effect are related to the chemical potential of the free system. Further, those influences relate to the density of particle number. As for the energy , when the chemical potential of the free system is odd /even, the absolute value of the relativistic effect

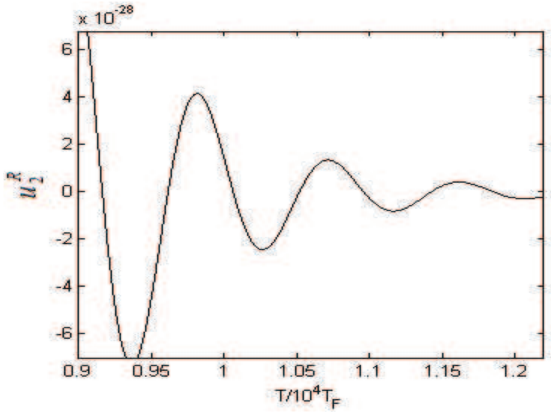


Figure 11: The curves of chemical potential u_2^R versus temperature.

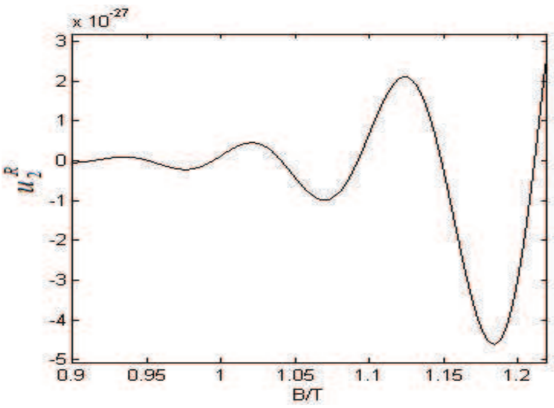


Figure 12: The curves of chemical potential u_2^R versus magnetic field.

increases/decreases with temperature increasing. When it comes to the system's chemical potential, if it is odd, with increasing temperature, the relativistic effect above turn to negative from positive; if even, with the increasing temperature, the absolute value of becomes smaller and the relativistic effect is negative. The relativistic effects on heat capacity and chemical potential is stronger and its oscillation also becomes more obviously with increasing of the magnetic field; those effects on energy are negative and stronger due to increasing of the magnetic field, however, does not show oscillating characteristic. At high temperatures, the amplitude of the relativistic effect decreases evidently with the increasing temperature, and increases evidently with the strengthening magnetic field.

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