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Preparation of atomic entangled states and Schrödinger cat states for N trapped ions driven by frequency-modulated laser

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Abstract. We propose a new scheme to prepare the Greenberger-Horne-Zeilinger states and atomic Schrödinger cat states for N trapped ions This scheme is based on the interaction of N trapped ions with a frequensy-modulated traveling wave light field. Preparations of these states can be all complished by one-step operation.

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Key words: trapped ion, entanglement state, atomic Schrödinger cat, frequency modulation

1 Introduction

Quantum entanglement has been enjoyed considerable attention in the last few years not only because it is fundamental in quantum mechanics but also because it plays a crucial role in quantum computation[1,2] and quantum teleportation[3].Various quantum systems have been suggested for the generation of entanglement such as trapped ions[4], nuclear magnetic resonance (NMR)[5], quantum dots[6], cavity quantum electrodynamics(CQED)[7] and others. In trapped ions system, pairs of hyperfine ground states provide an ideal host for quantum bits owing to less decoherence.. In order to entangle N trapped ions, the interaction between the ions is required and external control of this interaction is necessary to generate specific entanglement states[8]. For example, Many proposals are based on the interaction of optical Raman fields with the trapped ions[9,10,11,12]. However, these techniques require two or more laser beams acting on the trapped ions.

In this paper, we propose a scheme to prepare entanglement states and atomic Schrödinger cat states for N trapped ions using a single of frequency-nodulated laser. In our scheme, preparation of these states can be complished by one-step operation and a beam of laser is only required.

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2 Theoretical description

We consider N two-level ions with energy difference $\hbar\omega_\alpha$, which are trapped in a harmonic potential trap and interacts with a frequency-modulated traveling wave light field the Hamiltonian of the system can be written as $(\hbar=1)$[13]

$$\begin{aligned} H &= H_0 + H_I \\ H_0 &= \nu\alpha^\dagger\alpha + \frac{\omega_\alpha}{2}S_z \\ H_I &= \frac{\Omega}{2}\{e^{-i\omega_0 t - i\lambda\sin(\omega_p t+\varphi)}e^{ik_L x}S_+ + h.c\} \\ &= \frac{\Omega}{2}\{e^{-i\omega_0 t - i\lambda\sin(\omega_p t+\varphi)}e^{i\eta(\alpha+\alpha^+)}S_+ + h.c\} \end{aligned} \tag{1}$$

where α^+ and α are the corresponding creation and annihilation operators of center-of-mass vibrational quanta, $S_z=\sum_{j=1}^{N}S_{zj}$, $S_x=\sum_{j=1}^{N}S_{xj}$, $\sigma_{zj}=|e_j><e_j|-|g_j><g_j|$ and $\sigma_{xj}=|e_j><g_j|+|g_j><e_j|$ are Pauli operators for the j-th ion; Ω is the Rabi frequency; ω_0 is the carrier frequency, k_L wave vector; $\eta=k_L\sqrt{\hbar/2m\nu}$ is the Lamb-Dicke parameter, m mass of the ion, ν is the trapping frequency; $x=a^+ + a$ denotes a dimensionless position operator of the ion; φ is modulating phase, here we select $\varphi=\pi$; λ and ω_p is the modulating amplitude and the modulating frequency., respectively. Applying optical rotating wave approximation, we obtain

$$\begin{aligned} H_0 &= \nu\alpha^\dagger\alpha + \frac{\Delta}{2}S_z \\ H_I &= \frac{\Omega}{2}\{e^{+i\lambda\sin(\omega_p t)}e^{i\eta(\alpha+\alpha^+)}S_+ + h.c\} \\ &= \frac{\Omega}{2}\{\sum_m J_m(\lambda)e^{im\omega_p t}e^{i\eta(\alpha+\alpha^+)}S_+ + h.c\} \end{aligned} \tag{2}$$

where $\Delta=\omega_\alpha-\alpha_0$ is the detuning between the carrier frequency of light field and ionic transition frequency, $J_m(\lambda)$ is Bessel's function. In the following we select the detuning quantity $\Delta=0$. In the interaction picture, we thus obtain

$$H_I(t) = \frac{\Omega}{2}\{\sum_m J_m(\lambda)e^{im\omega_p t}e^{i\eta(\alpha e^{-i\nu t}+\alpha^+ e^{i\nu t})}S_+ + h.c\}. \tag{3}$$

Making Lamb-Dicke approximation, selecting that the modulating frequency satisfies $\nu-\omega_p \ll \omega_p,\nu$, and neglecting the fast oscillating terms, we can obtain

$$H_I(t) = i\varepsilon_0 S_x + i\varepsilon S_x(\alpha e^{-i\delta t} - \alpha^+ e^{i\varepsilon t}) \tag{4}$$

where $\delta=v-\omega_p$, $\varepsilon_0=\Omega J_0(\lambda)/2$, $\varepsilon=\eta\Omega J_1(\lambda)/2$. According to the definition of the displacement operator, during the infinitesimal interval $[t,t+dt]$, the corresponding evolution of the system is decided by

$$U(dt)=e^{-idtH_1(t)}=e^{-idt\varepsilon_0 S_x}e^{[d\alpha\alpha^+-d\alpha^*\alpha]S_x}=e^{-it\varepsilon_0 S_x}D(d\alpha S_x) \tag{5}$$

where $d\alpha=-\varepsilon e^{i\delta t}$. Applying formulae about the displacement operators, the evolution of the system in an interacting time t can be expressed as[6]

$$U(t)=e^{-idt\varepsilon_0 S_x}e^{i\vartheta(t)S_x^2}D(\int_\gamma d\alpha) \tag{6}$$

where

$$\vartheta(t)=Im\{\int_\gamma \alpha^* d\alpha\}=\frac{2\varepsilon_1^2}{\delta}(t-\frac{\sin\delta t}{\delta}) \tag{7}$$

$D(\int\limits_\gamma d\alpha)$ a the displacement operator. For a closed path, $D(\int\limits_\gamma d\alpha)$ can be represented as

$$D(\int_\gamma d\alpha)=D(0)=1 \tag{8}$$

We thus obtain he evolution operator of the system as

$$U(t)=e^{-i\varphi(t)S_x}e^{i\vartheta(t)S_x^2} \tag{9}$$

where $\varphi(t)=\varepsilon_0 t$.

3 Preparation of GHZ states for N atoms

We start with preparation of GHZ atomic states. We assume that the N trapped ions are initially located in the ground state, i.e., $|\Psi(0)>=|g_1g_2\cdots g_N>$. Using the representation of the operator S_z, the atomic states $|g_1g_2\cdots g_N>$ and $|e_1e_2\cdots e_N>$ can be expressed as $|g_1g_2\cdots g_N>=|N/2,-N/2>$, $|e_1e_2\cdots e_N>=|N/2,-N/2>$. On the other hand, we denote eigenstates of S_x as $|N/2,M>_x$, the atomic states $|N/2,-N/2>$ and $|N/2,N/2>$ can be expanded in terms of $|N/2,M>_x$ as[6]

$$|N/2,-N/2>=\sum_{M=-N/2}^{N/2} C_M|N/2,M>_x, \tag{10}$$

$$|N/2,N/2>=\sum_{M=-N/2}^{N/2} C_(-1)^{N/2-M}|N>2,M>_x. \tag{11}$$

Thus, the state vector of the system after an interacting time t is

$$|\Psi(t)>=U(t)|\Psi(0)>=\sum_{M=-N/2}^{N/2} C_M e^{i\vartheta(t)M^2}e^{-i\varphi(t)M}|N/2,M>_x \tag{12}$$

When N is even, M is an integer, selecting the modulating frequency ω_p and modulating amplitude λ to satisfy $\vartheta(t)=\pi/2$ and $\varphi(t)=\pi$, we obtain

$$\begin{aligned} |\Psi(t)>&=\sum_{M=-N/2}^{N/2} C_M[(-1)^M e^{-i\pi/4}+e^{i\pi/4}]|N/2,M>_x \\ &=\frac{1}{\sqrt{2}}(e^{i\pi/4}|g_1g_2\cdots g_N>+e^{-i\pi/4}|e_1e_2\cdots e_N>) \end{aligned} \tag{13}$$

On the other hand, When N is odd, M is an half-integer, choosing $\vartheta(t)=\pi/2$ and $\varphi(t)=3\pi/2$ by adjusting the parameters λ and ω_p, we then obtain

$$\begin{aligned} |\Psi(t)>&=\frac{1}{\sqrt{2}}\sum_{M=-N/2}^{N/2} C_M e^{i\pi/8}[1+(-1)^M]|N/2,M>_x \\ &=\frac{1}{\sqrt{2}}e^{i\pi/8}(|g_1g_2\cdots g_N>-(1)^{-N/2}|e_1e_2\cdots e_N>) \end{aligned} \tag{14}$$

From (13) and (14), we can see that N trapped ions are prepared in the Greenberger-Horne-Zeilinger states.

The proposed schemes for production of the GHZ states take in general two lasers in order to realize optical Raman interaction[1,11].The present scheme for production of the GHZ states has some advantages: (1) this method is very simple, because it only takes one step; (2) this method is realized easily in the present experiment because collimate adjustment of two or more laser beams can be avoided; (3) the entanglement is produced between N trapped ions.

4 Preparation of atomic Schrödinger cat states

The author of Ref.[14] investigated the properties of the atomic Schrödinger cat states, and proposed a scheme for preparation of this state based on interaction of field with atoms in a dispersive cavity. Here we propose a scheme for generation of the atomic Schrödinger cat states based on interation of trapped ions with frequency-modulated laser. We consider N two-level ions which are trapped in a harmonic potential trap driven by a beam of frequency-modulated laser The evolution of this system is governed by evolution operator (9). Here we select $J_0(\lambda)=0$, The evolution of this system then is

$$U(t)=e^{i\vartheta(t)S_x^2} \tag{15}$$

We assume that N trapped ions in a harmonic trap are prepared initially in the atomic coherent state[14]

$$|\Psi(0)>\equiv|\theta,\phi>=\sum_{k=0}^{N}\sqrt{\frac{N!}{(N=k)!k!}}e^{ik\phi}\times\sin^{N-k}(\frac{\theta}{2})\cos^k(\frac{\theta}{2})|N/2,N/2-k> \tag{16}$$

The state vector of the system after an interacting time t is

$$|\Psi(t)>=U(t)|\Psi(0)>=e^{i\vartheta(t)S_x^2}|\Psi(0)>=e^{i\pi S_y/2}e^{i\vartheta(t)S_z^2}e^{i\pi S_y/2}|\theta,\phi> \tag{17}$$

Noticing that

$$e^{i\pi S_y/2}|\theta,\phi>=\sum_{k=0}^{N}\sqrt{\frac{N!}{(N-k)!k!}}e^{i(N\phi+\pi)}e^{-ik\phi}\times\cos^{N-k}(\frac{\theta}{2})\sin^k(\frac{\theta}{2})|N/2,N/2-k>, \tag{18}$$

we can obtain

$$\begin{aligned}|\Psi(t)>=&e^{-i\pi S_y/2}\sum_{k=0}^{N}\sqrt{\frac{N!}{(N=k)!k!}}e^{i(N\phi+\pi)}e^{-ik\phi}\\&\times\cos^{N-k}(\frac{\theta}{2})\sin^k(\frac{\theta}{2})e^{i(N/2-k)^2\vartheta(t)}|N/2,N/2-k>,\end{aligned} \tag{19}$$

Select $\vartheta(t)=\pi/m$, m is a integer number, we gave

$$\begin{aligned}|\Psi(t)>=&e^{-i\pi S_y/2}\sum_{k=0}^{N}\sqrt{\frac{N!}{(N-k)!k!}}e^{-ik\phi'}\\&\times\cos^{N-k}(\frac{\theta}{2})\sin^k(\frac{\theta}{2})e^{-ik^2\pi/m}|N/2,N/2-k>,\end{aligned} \tag{20}$$

where $\phi'=\phi+N\pi/m-\pi$, $\phi_0=N\phi+N^2\phi/(4m)$. Now we use the expansion[14]

$$\exp\left\{\frac{i\pi}{m}k(k+1)\right\}=\sum_{q=0}^{m-1}f_q^{(0)}\exp\left\{\frac{i2\pi q}{m}k\right\} \tag{21}$$

$$\exp\left\{\frac{i\pi}{m}k^2\right\}=\sum_{q=0}^{m-1}f_q^{(e)}\exp\left\{\frac{i2\pi q}{m}k\right\} \tag{22}$$

for m odd and even, respectively along with the inversion relations

$$f_q^{(0)}=\frac{1}{m}\sum_{k=0}^{m-1}\exp\left\{-\frac{i2\pi q}{m}k\right\}\exp\left\{\frac{i\pi}{m}k(k+1)\right\} \tag{23}$$

$$f_q^{(e)}=\frac{1}{m}\sum_{k=0}^{m-1}\exp\left\{-\frac{i2\pi q}{m}k\right\}\exp\left\{\frac{i\pi}{m}k^2\right\} \tag{24}$$

When m is even, we can obtain

$$|\Psi(t)>=(-1)^Ne^{i\phi_0}\sum_{q=0}^{m-1}f_q^{(e)}e^{-iN(\phi-\pi+N\pi/m+2q\pi/m)}|\theta,\phi+(N+2q)\pi/m> \tag{25}$$

When m is odd, we have

$$|\Psi(t)>=(-1)^N e^{i\phi_0}\sum_{q=0}^{m-1} f_q^{(0)} e^{-iN(\phi-\pi+(N-1)\pi/m+2q\pi/m-\pi)}|\theta,\phi+(N+2q-1)\pi/m> \quad (26)$$

The expression (25) and (26) show that at the interaction time t, an atomic coherent states evolves into superposition of the atomic coherent states. For example, we consider the case of $m=2$, Eq. (25) is changed into

$$\begin{aligned}|\Psi(t)>=(-1)^N e^{i\phi_0'}\frac{\sqrt{2}}{2}[e^{i\pi/4}e^{-i(\phi+N\pi/2-\pi)N}|\theta,\phi+N\pi/2>\\ +e^{-i\pi/4}e^{-i(\phi+N\pi/2)N}|\theta,\phi+(N+2)\pi/2>]\end{aligned} \quad (27)$$

It is easily seen that this is an atomic Schrödinger cat state.

Compared with the scheme for production of the atomic Schrödinger cat state given in Ref. [14], present scheme is very simple and less decoherence.

5 Conclusion

We have investigated theoretically two new schemes to prepare nonclassical atomic states of the trapped ions, namely, GHZ atomic states and atomic Schrödinger cat for N ions. These schemes are based on the frequency-modulated of a traveling wave light field which the trapped ions are located in a harmonic potential trap. These schemes can be realized in the present experiment.

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