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Numerical Approximation and Error Analysis for the Timoshenko Beam Equations with Boundary Feedback

Fule Li* and Kaimei Huang
(*School of Science, Laiyang Agricultural College, Qingdao 266109, China*
E-mail: lfl2004666@126.com, hkm1966@163.com)

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Abstract

In this paper, the numerical approximation of a Timoshenko beam with boundary feedback is considered. We derived a linearized three-level difference scheme on uniform meshes by the method of reduction of order for a Timoshenko beam with boundary feedback. It is proved that the scheme is uniquely solvable, unconditionally stable and second order convergent in L_∞ norm by using the discrete energy method. A numerical example is presented to verify the theoretical results.

Keywords: Timoshenko beam; boundary feedback; partial differential equation; finite difference; solvability; convergence; stability.
Mathematics subject classification: 65M06, 65M12, 65M15

1. Introduction

In this paper, we study the numerical approximation of a Timoshenko beam with boundary feedback. As already pointed out by Xu and Feng [12], the boundary control problem of flexible structure has recently attracted much attention with the rapid development of high technology such as space science and flexible robots. A number of authors (see [6, 7, 12–14, 17]) have considered control problems associated with the Timoshenko beam and obtained many interesting results. At the same time, the finite element method and the finite difference method are effectively applied to the Timoshenko beam for solving the boundary stabilization and the numerical solution (see [1–3, 5, 7, 8]). In particular, Li and Sun [7] studied the numerical solution of a Timoshenko beam problem with boundary feedback, given by [12]

$$\rho\frac{\partial^2 w(x,t)}{\partial t^2}-K\left[\frac{\partial^2 w(x,t)}{\partial x^2}-\frac{\partial\varphi(x,t)}{\partial x}\right]=f_1(x,t),\quad 0<x<l,\ t>0, \tag{1.1}$$

$$I_\rho\frac{\partial^2\varphi(x,t)}{\partial t^2}-EI\frac{\partial^2\varphi(x,t)}{\partial x^2}-K\left[\frac{\partial w(x,t)}{\partial x}-\varphi(x,t)\right]=f_2(x,t),\quad 0<x<l,\ \ t>0, \tag{1.2}$$

$$w(0,t)=0,\quad \varphi(0,t)=0,\quad t>0, \tag{1.3}$$

*Corresponding author.

$$\frac{\partial w(l,t)}{\partial x} - \varphi(l,t) = -\alpha\frac{\partial w(l,t)}{\partial t} + g_1(t), \ \ \frac{\partial \varphi(l,t)}{\partial x} = -\beta\frac{\partial \varphi(l,t)}{\partial t} + g_2(t), \quad t>0, \tag{1.4}$$

$$w(x,0) = g_3(x), \ \frac{\partial w(x,0)}{\partial t} = g_4(x), \ \ \varphi(x,0) = g_5(x), \ \frac{\partial \varphi(x,0)}{\partial t} = g_6(x), \quad 0 \le x \le l, \tag{1.5}$$

where ρ, I_ρ, EI, K, l are mass density, moment of mass inertia, rigidity coefficient, shear modulus of elasticity and length of the beam, respectively, and α, β are given positive gain feedback constants, $w(x,t)$ is the transversal displacement and $\varphi(x,t)$ is the rotational angle of the beam. The boundary conditions in (1.3) and (1.4) mean that the beam is clamped at $x=0$ and controlled at $x=l$ by the force and moment feedback. Li and Sun derived a finite difference scheme by the method of reduction of order. It is proved by the discrete energy method that the scheme is uniquely solvable, unconditionally stable and second order convergent in L_∞ norm.

In this paper, we consider finite difference simulation for the following Timoshenko beam equations with boundary feedback [4,15]

$$\rho\frac{\partial^2 w(x,t)}{\partial t^2} - K\left[\frac{\partial^2 w(x,t)}{\partial x^2} - \frac{\partial \varphi(x,t)}{\partial x}\right] = f_1(x,t), \quad 0<x<l, \ t>0, \tag{1.6}$$

$$I_\rho\frac{\partial^2 \varphi(x,t)}{\partial t^2} - EI\frac{\partial^2 \varphi(x,t)}{\partial x^2} - K\left[\frac{\partial w(x,t)}{\partial x} - \varphi(x,t)\right] = f_2(x,t), \quad 0<x<l, \ \ t>0, \tag{1.7}$$

$$w(0,t)=0, \quad \varphi(0,t)=0, \quad t>0, \tag{1.8}$$

$$\frac{\partial w(l,t)}{\partial x} - \varphi(l,t) = -\alpha\frac{\partial^2 w(l,t)}{\partial t^2} + g_1(t), \ \ \frac{\partial \varphi(l,t)}{\partial x} = -\beta\frac{\partial^2 \varphi(l,t)}{\partial t^2} + g_2(t), \quad t>0, \tag{1.9}$$

$$w(x,0) = g_3(x), \ \frac{\partial w(x,0)}{\partial t} = g_4(x), \ \varphi(x,0) = g_5(x), \ \frac{\partial \varphi(x,0)}{\partial t} = g_6(x), \quad 0 \le x \le l, \tag{1.10}$$

where $\rho, I_\rho, EI, K, l, \alpha, \beta$ have the same sense as in (1.6)-(1.10). Zietsman et al. [17] investigated the efficiency and accuracy of the finite element method for calculating the eigenvalues and eigenmodes. Hou et al. [4], Yan and Feng [15,16] studied the stabilization problem of the coupled nonuniform Timoshenko beam using boundary feedback.

The difference of the equations in (1.1)-(1.5) and in (1.6)-(1.10) is the boundary condition at $x=l$. In (1.4), the maximal order of the derivatives with respect to time variable t is one, which is less than the maximal order of the derivatives in (1.9).

Throughout this article, we assume that $f_1(x,t), f_2(x,t) \in C([0,l]\times[0,T])$, $g_1(t), g_2(t) \in C[0,T], g_3(x), g_4(x), g_5(x), g_6(x) \in C[0,l]$ and $g_3(0) = g_5(0) = 0$ such that problem (1.6)-(1.10) has a smooth solution $\{w(x,t) \in C^{4,3}([0,l]\times[0,T]), \varphi(x,t) \in C^{4,3}([0,l]\times[0,T])\}$.