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SOME RESEARCHES ON WEAK CONVERGENCE OF KERGIN INTERPOLATION *

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Abstract *The aim of this paper is to study the weak integral convergence of Kergin interpolation. The results of the weighted integral convergence and the weighted (partial) derivatives integral convergence of Kergin interpolation polynomial for the smooth functions on the unit disk were obtained in the paper. Those generalized Liang's main results were acquired in 1998 to the more extensive situation. At the same time, the estimation of convergence rate of Kergin interpolation polynomial is given by means of introducing a new kind of smooth norm.*

Key words *Kergin interpolation, weak convergence, convergence order.*

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1 Introduction

P.Kergin [1] proposed a new kind of multivariate interpolation operator, which is known as Kergin interpolation. In the same year, C.A.Micchelli [2], [3] gave a Newton representation of the operator, and basing on this representation, the error estimation of Kergin interpolation was established. Also by using this representation, L.Bos [4] and X.Z.Liang [5] researched the convergence of Kergin interpolation for the analytic functions on the unit disk. On the other hand, by the Lagrange representation of Kergin interpolation, X.Z Liang [8] discussed the uniform convergences of Kergin interpolation and its derivatives for the smooth functions on the unit disk. The aim of this paper is to research weak integral convergence of Kergin interpolation on the disk.

Let D denote the unit disk in real plane $\mathbf{R}^2$, i.e.,

$$D = \{X = (x, y)^T \in \mathbf{R}^2 \mid x^2 + y^2 \leq 1\},$$

$C^l(D)$ denote the space of all l-time continuously differentiable functions defined in D, $\mathbf{P}_m^2$ denote the space of all bivariate real coefficient polynomials of total degree $\leq m$, and $n \geq 2$ be natural

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number. Set

$$\begin{cases} \theta_i = \dfrac{2i\pi}{n}, \\ X^i = (\cos\theta_i, \sin\theta_i)^T, \end{cases} \qquad 1 \le i \le n,$$

$$n_{jk} = n_{kj} = (\cos\frac{1}{2}(\theta_j+\theta_k), \sin\frac{1}{2}(\theta_j+\theta_k))^T, \qquad 1 \le j < k \le n.$$

For $f(X) \in C^0(D)$, and $X, Y \in \mathbf{R}^2$, we introduce integral functional as follows:

$$\int_{[X,Y]} f = \int_0^1 f(\lambda X + (1-\lambda)Y)\mathrm{d}\lambda.$$

Also let

$$\begin{aligned} & T_m(t) = \cos(m \text{ arccos} t),\ U_m(t) = T_m'(t), \\ & R_m(t) = \frac{T_m(t)+T_{m-1}(t)}{t+1}, \quad S_m(t) = \frac{T_m(t)-T_{m-1}(t)}{t-1}, \\ & X = (x,y)^T = (r\cos\theta, r\sin\theta)^T, \\ & t_i = r\cos\left(\frac{i\pi}{n} - \theta\right) = \cos\frac{i\pi}{n}x + \sin\frac{i\pi}{n}y, \qquad i = 0,1,2,\cdots. \end{aligned}$$

On the basis of the results in [6], Liang gave the following theorem:

Theorem A [7] For any $f(X) \in C^1(D)$, there exists a unique polynomial $K_n(X) = K_n(f;X) \in \mathbf{P}^2_{n-1}$, such that the following conditions are satisfied

$$\begin{cases} K_n(X^i) = f(X^i), & 1 \le i \le n, \\ \displaystyle\int_{[X^j,X^k]} \frac{\partial}{\partial n_{jk}}(K_n - f) = 0, & 1 \le j < k \le n. \end{cases}$$

Here $K_n(X) = K_n(f;X)$, which is named as Kergin interpolation polynomial of $f(X)$, has the following representation

$$K_n(X) = \sum_{j=1}^n f(X^j)L_j(X) + \sum_{1\le j<k\le n} L_{jk}(X)\int_{[X^j,X^k]} \frac{\partial}{\partial n_{jk}} f.$$

Where

(I) If n is even (set $n = 2m$), then

$$L_{jk}(X) = \begin{cases} \dfrac{\sin^2\frac{k-j}{n}\pi}{m^2} \cdot \dfrac{(T_m(t_{j+k}))^2}{t_{j+k} - \cos\frac{k-j}{n}\pi}, & \text{if } k-j \text{ is odd;} \\ \dfrac{\sin^2\frac{k-j}{n}\pi}{m^4} \cdot \dfrac{(1-t^2_{j+k})(U_m(t_{j+k}))^2}{t_{j+k} - \cos\frac{k-j}{n}\pi}, & \text{if } k-j \text{ is even.} \end{cases}$$

$$L_j(X) = \sum_{i=1}^m \frac{\cos\frac{i\pi}{m}}{2m^4} \cdot \frac{(t^2_{2i+2j}-1)(U_m(t_{2i+2j}))^2}{t_{2i+2j} - \cos\frac{i\pi}{m}} - \sum_{i=1}^m \frac{\cos\frac{2i-1}{2m}\pi}{2m^2} \cdot \frac{(T_m(t_{2i+2j-1}))^2}{t_{2i+2j-1} - \cos\frac{2i-1}{m}\pi};$$

(II) If n is odd (set $n = 2m-1$), then

$$L_{jk}(X) = \begin{cases} \dfrac{2\sin^2\frac{k-j}{n}\pi}{n^2} \cdot \dfrac{(1+t_{j+k})(R_m(t_{j+k}))^2}{t_{j+k} - \cos\frac{k-j}{n}\pi}, & \text{if } k-j \text{ is odd,} \\ \dfrac{2\sin^2\frac{k-j}{n}\pi}{n^4} \cdot \dfrac{(1-t_{j+k})(S_m(t_{j+k}))^2}{t_{j+k} - \cos\frac{k-j}{n}\pi}, & \text{if } k-j \text{ is even.} \end{cases}$$