East Asian Journal on Applied Mathematics
doi: 10.4208/eajam.181016.300517b

Vol. **7**, No. 4, pp. 728-740
November 2017

A Coupled Model for Wave Run-up Simulation

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Received 18 October 2016; Accepted (in revised version) 30 May 2017.

Abstract. Simplified models like the shallow water equations (SWE) are commonly adopted for describing a wide range of free surface flow problems, like flows in rivers, lakes, estuaries, or coastal areas. In the literature, numerical methods for the SWE are mostly mesh-based. However, this macroscopic approach is unable to accurately represent the complexity of flows near coastlines, where waves nearly break. This fact prompted the idea of coupling the mesh-based SWE model with a meshless particle method for solving the Euler equations. In a previous paper, a method to couple the staggered scheme SWE and the smoothed particle hydrodynamics (SPH) Euler equations was developed and discussed. In this article, this coupled model is used for simulating solitary wave run-up on a sloping beach. The results show strong agreement with the experimental data of Synolakis. Simulations of wave overtopping over a seawall were also performed.

AMS subject classifications: 76M28, 76M10, 76B25

Key words: Smoothed particle hydrodynamics, staggered conservative scheme, solitary wave run-up, wave overtopping.

1. Introduction

Determination of wave run-up characteristics and possible flooding scenarios are important factors in the design of coastal protection structures. In recent years, wave run-up has been investigated experimentally, theoretically and numerically. For experimental studies, one can consult [16, 17], for instance.

The shallow water equations are a widely used model for describing wave run-up characteristics. Grid-based numerical schemes solving the shallow water equations are abundant in the literature. However, wave run-up is a physically complex phenomenon, especially when it involves wave breaking. In this paper, we combine a numerically efficient grid-based scheme with a meshless SPH method. The coupling procedure between staggered grid SWE scheme and the SPH computations developed in [4] is used here. The

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staggered scheme is implemented on a large flat domain, whereas SPH computations focus on the shoreline or close to off-shore structures, where a complex description of the free-surface is required.

In research related to wave run-up, the experimental data measuring solitary wave run-up on a sloping beach as recorded by Synolakis in [16, 17] serves as the canonical benchmark test. Here, we present simulations of solitary waves climbing up a sloping beach calculated using three different numerical scenarios: the staggered SWE scheme, the SPH method, and the coupled model. For each case, the run-up height is recorded and compared with the aforementioned experimental data.

The outline of this paper is as follows: In Section 2, governing equations for free surface flows (the shallow water equations and the Euler equations), along with their corresponding numerical methods (a staggered grid approach and an SPH approach, respectively), are reviewed. Moreover, a coupled model of SWE and Euler models is discussed. In Section 3, simulation results for solitary wave run-up on a sloping beach are computed using each of these models. Section 4 focuses on an application of the coupled model to simulate wave overtopping of a seawall.

2. Numerical Models

In this section, we discuss three models: the staggered grid scheme for the shallow water equations model, the SPH method for the Euler model, and a coupled model combining those two methods.

2.1. Shallow water model

Consider the following shallow water model, which holds for relatively long waves in a shallow region:

$$h_t + (hu)_x = 0, \tag{2.1}$$

$$u_t + uu_x + gh_x = 0. \tag{2.2}$$

In the above equations, $u(x,t)$ denotes the depth-averaged horizontal velocity, $h(x,t)$ denotes the total water layer thickness such that $h(x,t) = \eta(x,t) + d(x)$, where $\eta(x,t)$ denotes the free surface elevation and $d(x)$ describes the depth of the bottom topography relative to the undisturbed free surface height (Fig. 1). Here, we consider a staggered conservative scheme for solving the above shallow water equations, for which a discrete formulation of (2.1)–(2.2) is given by

$$\frac{h_i^{n+1} - h_i^n}{\Delta t} = -\left(\frac{{}^*h^n_{i+\frac{1}{2}} u^n_{i+\frac{1}{2}} - {}^*h^n_{i-\frac{1}{2}} u^n_{i-\frac{1}{2}}}{\Delta x}\right), \tag{2.3}$$

$$\frac{u^{n+1}_{i+\frac{1}{2}} - u^n_{i+\frac{1}{2}}}{\Delta t} = -\frac{1}{\overline{h}_{i+\frac{1}{2}}}\left(\frac{\overline{q}_{i+1}\,{}^*u_{i+1} - \overline{q}_i\,{}^*u_i}{\Delta x} - u_{i+\frac{1}{2}}\frac{\overline{q}_{i+1} - \overline{q}_i}{\Delta x}\right) - g\left(\frac{\eta^{n+1}_{i+1} - \eta^{n+1}_i}{\Delta x}\right), \tag{2.4}$$