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Hybrid Variational Model for Texture Image Restoration

Liyan Ma[1,*], Tieyong Zeng[2,3] and Gongyan Li[1]

[1] *Institute of Microelectronics of Chinese Academy of Sciences, Beijing, China.*
[2] *Department of Mathematics, Hong Kong Baptist University, Kowloon Tong, Hong Kong, China.*
[3] *HKBU Institute of Research and Continuing Education, Shenzhen Virtual University Park, Shenzhen 518057, China.*

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Abstract. The hybrid variational model for restoration of texture images corrupted by blur and Gaussian noise we consider combines total variation regularisation and a fractional-order regularisation, and is solved by an alternating minimisation direction algorithm. Numerical experiments demonstrate the advantage of this model over the adaptive fractional-order variational model in image quality and computational time.

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1. Introduction

Image restoration is a crucial pre-processing step for many applications, and it is widely used in computer vision. In order to improve image quality, various models have been proposed recently. Image restoration is a typical inverse problem that can be viewed as an estimation problem. Here we focus on recovering images corrupted by a spatially-invariant blur and a Gaussian noise. The image degradation model can be represented by the equation

$$g = Af + n\,, \tag{1.1}$$

where $f \in \mathbb{R}^{N_y \times N_x}$ is a clear image, $g \in \mathbb{R}^{N_y \times N_x}$ is the observed image, A is a known linear operator from $\mathbb{R}^{N_y \times N_x}$ to $\mathbb{R}^{N_y \times N_x}$, and n represents an additive Gaussian noise.

Inverse problems in image processing are typically ill-posed. A common strategy to tackle such problems is a variational approach, which we also use here. There are two phases in a variational model — viz. regularisation and data fidelity. The regularisation plays an important role in inverse problems, and is used to reduce the space of possible

*Corresponding author. *Email addresses:* `maliyan@ime.ac.cn` (L. Ma), `zeng@hkbu.edu.hk` (T. Zeng), `ligongyan@ime.ac.cn` (G. Li)

solutions. A proper regularisation has to capture the most important properties of an image and improve the visual quality of the recovered image. Over the past decades, many works deal with the regularisation procedure — cf. Refs. [7, 13, 17, 24, 36, 37]. A common approach used in inverse problems is the Tikhonov L_2-norm regularisation [35] that effectively removes the noise, but since it over-smooths image edges Rudin *et al.* [32] proposed an L_1-norm regularisation called total variation (TV) regularisation that is convex, preserves sharp edges and allows the use of powerful optimisation methods [3, 5, 22, 41]. The TV regularisation has been successfully applied in applications [11, 18, 25, 40] such as image restoration, image segmentation, motion estimation, and image reconstruction. However, it does not preserve fine details and has a staircase effect caused by the model assumption that the underlying image is piecewise constant, which in most cases does not hold.

To overcome disadvantages of the TV regularisation, several other regularisation functionals have been considered recently. Chambolle & Lions [10] introduced second order derivatives; Berkels *et al.* [4] studied an anisotropic regularisation based on the local structure of the image; a locally varying regularisation weight was used by Grasmair [21] in locally adaptive TV regularisation; Lou *et al.* [23] examined the image gradient and proposed a weighted difference of anisotropic and isotropic TV regularisation; and for a better balance between noise removal and edge preservation, You & Kaveh [42] introduced a fourth-order PDE model. Bredies *et al.* [6] proposed a total generalised variation (TGV) regularisation, which is a version of the TV regularisation involving higher-order derivatives that preserves piecewise polynomial intensities and sharp edges, suppressing the staircase effect and improving the visual image quality. Setzer *et al.* [33] reduced the staircasing artifacts of the restoration results in Chambolle & Lions [10] by an infimal convolutions modification. Bai & Feng [2] developed a fractional-order anisotropic diffusion model as a replacement for second order and fourth-order anisotropic diffusion equations; and using the regularisation parameter selection method of Ref. [19], Zhang *et al.* [46] proposed an adaptive fractional-order multi-scale method for image denoising. To fix texture deblurring, Chan *et al.* [12] introduced a spatially adaptive fractional order TV regularisation term with different fractional orders for different texture regions, which produces very promising results on texture images but finding the texture map is not easy. Regularisation with different fractional orders utilises many neighbourhood pixels which can cause unexpected errors, especially for pixels on the boundary between different textures.

While computing the partial derivatives in fractional-order regularisation, the many neighbourhood pixels involved effectively collect texture information — and here we focus on texture image deblurring based on a fractional order TV regularisation. To overcome the limitation of the method in Ref. [12], we use an hybrid variational model that combines TV regularisation and fractional order TV regularisation, so time consuming texture map calculations are avoided. Experimental results demonstrate the efficiency of this approach, in both the quality of image restoration and computational time. In Section 2, we recall the concept of fractional-order regularisation, and our algorithm is introduced in Section 3. Section 4 contains numerical results demonstrating the algorithm efficiency, and our concluding remarks are in Section 5.