Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2016-0183

Vol. **22**, No. 2, pp. 441-459
August 2017

# DNS Study on Vortex and Vorticity in Late Boundary Layer Transition

Yiqian Wang[1,2], Yong Yang[1], Guang Yang[1,3] and Chaoqun Liu[1,*]

[1] *Department of Mathematics, University of Texas at Arlington, Arlington, TX 76019, USA.*
[2] *School of Aerospace Engineering, Tsinghua University, Beijing 100084, China.*
[3] *Beihang University, Beijing 100191, China.*

Received 27 October 2016; Accepted (in revised version) 21 December 2016

**Abstract.** Vortex and vorticity are two correlated but fundamentally different concepts which have been the central issues in fluid mechanics research. Vorticity has rigorous mathematical definition (curl of velocity), but no clear physical meaning. Vortex has clear physical meaning (rotation) but no rigorous mathematical definition. For a long time, many people treat them as a same thing. However, based on our high-order direct numerical simulation (DNS), we found that first, ”vortex” is not ”vorticity tube” or ”vortex tube” which is widely defined as a bundle of vorticity lines without any vorticity line leak. Actually, vortex is an open area for vorticity line penetration. Second, vortex is not necessarily congregation of vorticity lines, but dispersion in many 3-dimensional cases. Some textbooks say that vortex cannot end inside the flow field but must end on the solid wall (and/or boundaries). Our DNS observation and many other numerical results show almost all vortices are ended inside the flow field. Finally, a more theoretical study shows that neither vortex nor vorticity line can attach to the solid wall and they must be detached from the wall.

**AMS subject classifications**: 76F06, 76F40, 76M23, 76D17

**Key words**: Vortex, vorticity, vorticity line, vorticity tube, Λ-vortex, hairpin vortex, DNS, flow transition, turbulence.

## Nomenclature

$M_\infty =$ Mach number
Re $=$ Reynolds number
$\delta_{in} =$ inflow displacement thickness

*Corresponding author. *Email addresses:* `yqwang@nuaa.edu.cn` (Y. Wang), `yong.yang@mavs.uta.edu` (Y. Yang), `guang.yang@uta.edu` (G. Yang), `cliu@uta.edu` (C. Liu)

 

$T_w =$ wall temperature
$T_\infty =$ free stream temperature
$Lz_{in} =$ height at inflow boundary
$Lz_{out} =$ height at outflow boundary
$Lx =$ length of computational domain along $x$ direction
$Ly =$ length of computational domain along $y$ direction
$x_{in} =$ distance between leading edge of flat plate and upstream boundary of computational domain
$A_{2d} =$ amplitude of 2D inlet disturbance
$A_{3d} =$ amplitude of 3D inlet disturbance
$\omega =$ frequency of inlet disturbance
$\alpha_{2d}, \alpha_{3d} =$ two and three dimensional streamwise wave number of inlet disturbance
$\beta =$ spanwise wave number of inlet disturbance
$R =$ ideal gas constant
$\gamma =$ ratio of specific heats
$\mu_\infty =$ viscosity
$x,y,z =$ streamwise, spanwise, normal directions

## 1 Introduction

Vorticity has rigorous mathematical definition (curl of velocity), but no clear physical meaning. On the other hand, vortex has clear physical meaning (rotation) but no rigorous mathematical definition. For a long time, many people treat them as a same thing.

Vorticity are ubiquitously seen in nature, ranging from smoke rings to clouds, from bubble rings to hurricanes, from swirls in a washing pool to whirlpools in the sea. It is intuitively acknowledged that a vortex is a fluid region with rotational motions. However, the problem of giving a precise and rational vortex definition is still an open issue [1,2], even though people's understanding towards vortices has been greatly deepened ever since. Existing vortex identification criteria including $\lambda_2$, $Q$ and $\tilde{\Delta}$ methods [3–5] have proved to be efficient in visualizing vortical structures. Nevertheless, these methods still fail to give a quantitative definition of vortices. Thus, a better understanding of vortices and a rational definition become more and more pressing since the discovery of coherent vortical structures and their key role in transitional and turbulent flows. Recently, a new vortex identification method called $\Omega$-method is proposed by Liu et al. [6] based on the idea that vortices locate where vorticity overtake deformation and has advantages like no need for a case-related threshold, easiness to perform and the ability to capture strong and weak vortices at the same time.

Vorticity, on the other hand, is mathematically defined as the curl of velocity field.