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A High-Resolution Cell-Centered Lagrangian Method with a Vorticity-Based Adaptive Nodal Solver for Two-Dimensional Compressible Euler Equations

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Abstract. In this work, a second-order high-resolution LAgrangian method with a Vorticity-based Adaptive Nodal Solver (LAVANS) is proposed to overcome the numerical difficulty of traditional Lagrangian methods for the simulation of multi-dimensional flows. The work mainly include three aspects to improve the performance of the traditional CAVEAT-type cell-centered Lagrangian method. First, a vorticity-based adaptive least-squares method for vertex velocity computation is proposed to suppress nonphysical mesh distortion caused by the traditional five-point-stencil least-squares method. Second, a simple interface flux modification is proposed such that the geometry conservation law is satisfied. Third, a generalized Riemann problem solver is employed in the LAVANS scheme to achieve one-step time-space second-order accuracy. Some typical benchmark numerical tests validate the performance of the LAVANS scheme.

AMS subject classifications: 76M12, 76N15, 76T99

Key words: Cell-centered Lagrangian scheme, CAVEAT scheme, geometry conservation law, vertex velocity, generalized Riemann problem (GRP) solver.

1 Introduction

Lagrangian schemes can track material interfaces and free surfaces with natural high-resolution, thus they are playing an indispensable role in many engineering and scientific problems, such as inertial confined fusion [1–3], astrophysics, detonation and impact

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and so on. With increasingly more complicated application demands, robust and high-resolution Lagrangian methods are still posing a challenging task for the research on hydrodynamics numerical methods.

According to the location where physical variables are defined over computational cells, Lagrangian methods [4] can be classified into staggered schemes and cell-centered schemes. For a staggered scheme, the velocity variable is defined at vertices of computational cells and other physical variables such as density, pressure, and energy are defined at cell centers. For a cell-centered scheme, all physical variables are defined at cell centers. These two types of Lagrangian methods have their own advantages and disadvantages.

The staggered schemes employed in most hydro-codes have been remarkably successful over the past decades in solving complex multidimensional compressible fluid flows. In staggered schemes, a nodal solver can directly reflect the interaction of forces from its surrounded cells to track interfaces faithfully. Even with wide applications in many fields, the staggered methods have some inherent defects, such as the inconsistency between momentum and energy, the dependence of artificial viscosity and so on. In order to improve such aspects, cell-centered schemes became prosperous in recent years. Cell-centered schemes can maintaining consistency of discretization of momentum flux and that of total energy flux. And cell-centered schemes can be easily extended to achieve high-order accuracy. By choosing the cell-centered placement of all fluid variables as most Eulerian schemes do, many fully developed technologies employed in Eulerian methods can be extended to cell-centered Lagrangian methods. Whether the cell-centered schemes reflect the genuine interface positions compared with the staggered schemes are still a problem to be further studied. We believe that the problem is quite related to the multi-dimensional property and the computational accuracy of the nodal solver.

From the point of the different modeling of interface flux, cell-centered Lagrangian methods can also be classified into two types. One is based on the continuous assumption of the edge-flux, so there is a unique numerical flux defined at the cell interface. The CAVEAT-type scheme from the CAVEAT code [5] is belong to this type. It has been developed and used in some important application codes, e.g. the GALE code [13]. Another cell-centered scheme is based on the discontinuous assumption of the edge-flux, so there are more than one numerical fluxes defined at the cell interface. GLACE-type schemes [7–9, 19, 20] are of this type. As far as we know, seldom research is used to measure the extent of nodal velocity between simulation and reality. In this work, we are concerned with the above first type cell-centered schemes and try to improve its performance.

The main deficiency of CAVEAT-type schemes lies in the computation of multi-dimensional flows with large interface deformation. The deficiency is featured with spurious mesh tangling, which often result in computation blowing up and even nonphysical numerical outcome. In our opinion, the main reason of these problems is related to the nodal solver which is the key part of cell-centered Lagrangian methods. In the CAVEAT scheme, a five-point-stencil least-squares method is used to compute the nodal velocity.