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NONNEGATIVE MATRIX FACTORIZATION WITH BAND CONSTRAINT*

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Abstract

In this paper, we study a band constrained nonnegative matrix factorization (band NMF) problem: for a given nonnegative matrix Y, decompose it as $Y \approx AX$ with A a nonnegative matrix and X a nonnegative block band matrix. This factorization model extends a single low rank subspace model to a mixture of several overlapping low rank subspaces, which not only can provide sparse representation, but also can capture significant grouping structure from a dataset. Based on overlapping subspace clustering and the capture of the level of overlap between neighbouring subspaces, two simple and practical algorithms are presented to solve the band NMF problem. Numerical experiments on both synthetic data and real images data show that band NMF enhances the performance of NMF in data representation and processing.

Mathematics subject classification: 15A23, 65F30, 90C59.
Key words: Nonnegative matrix factorization, Band structure, Subspace clustering, Sparse representation, Image compression.

1. Introduction

In many large datasets, the relevant information often lies in a low dimensional subspace of the ambient space, which leads to a large interest in representing data with low rank approximations. Particularly, the nonnegative matrix factorization (NMF) for nonnegative data analysis, not only uncovers latent low dimensional structures intrinsic in high dimensional data but also provides a nonnegative, part-based representation of data. With these strengths, NMF has attracted intensive studies in the last decades and various factorization models, algorithms and regularized variants have been developed for different purposes and applications [1,2,5,9,20].

Despite the growing availability of tools for NMF, many techniques ignore an underlying information that the data often contains some type of structure that enables intelligent representation and processing. In computer vision, for example, a collection of images of an object taken under different illuminations has not only a low rank representation [4], but also significant spatial structure relating to the statistics of the scene, such as sparseness on a particular wavelet basis or low total variation [3]. Also, for monaural blind source separation [8], the coefficient matrix X with block diagonal structure indicates where each source is active when there is no training data for the individual sources.

In recent decade, more and more strategies have been given to represent and capture the intrinsic structure implied in the data. Kim et al. [13] introduced a novel formulation of sparse NMFs to get the sparse structure. Cai et al. [12] proposed a graph regularized nonnegative

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matrix factorization (GNMF) model to discover the intrinsic geometrical and discriminating structure of the data space by constructing a nearest neighbor graph. Pei et al. [14] incorporated neighbor isometric regularized constraint in the optimization of the NMF to extract the low rank space that preserves neighbor isometric geometry structure. Similar to these methods, Wu et al. [15] proposed a nonnegative low rank and group sparse matrix factorization (NLRGS) method to capture the grouping structure by simultaneously integrating low rank and group sparse constraints. All these methods, however, are difficult to clearly identify the implicit data structural characteristics for two reasons. First, all these methods are essentially based on the hypothesis that all data is approximately drawn from a low rank subspace. However, a given dataset can seldom be well described by a single subspace. A more reasonable model is to consider samples as a mixture of several overlapping low rank subspaces, as shown in Fig. 1.1 (left). The right is a real image whose pixels is continuously changing, so its pixels are well characterized by its neighbor points, which favors our proposed overlapping subspace model. Second, there are limitations of using regularization method to capture the data structural characteristics. Indeed, when data are from a union of five overlapping subspaces, almost all of the methods which extend the NMF problem formulation to include additional regularization terms on A and/or X cannot capture this overlapping low rank subspace structure (see Fig. 4.3(a-c)).

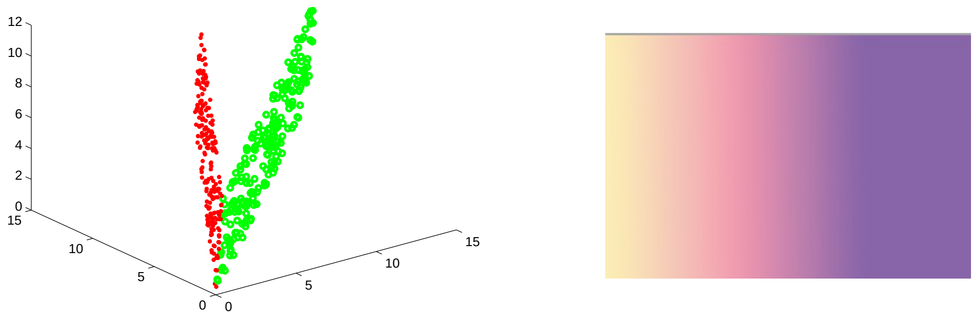

Fig. 1.1. A display of overlapping subspace model.

Besides, low rank representation (LRR) [10] and its various modified methods [16,17] incorporated the low rank constraint to represent each sample as a linear combination of other samples. However, these low rank methods neither derive the projection subspace of original examples nor get the block band structure of the coefficient matrix when data are overlapping. More specifically, LRR and its variants can only obtain the block diagonal structure of the coefficient matrix approximatively although some subspaces share a few bases (see Fig. 4.3(d)).

To overcome the aforementioned deficiencies, this paper proposes a band constrained nonnegative matrix factorization model (named band NMF). Particularly, band NMF extends a single low rank subspace model to a mixture of several overlapping low rank subspaces, which not only can provide sparse representation but also can capture significant grouping structure from a dataset. Additionally, the coefficient matrix X with band structure is also considered as a filtering matrix that performs continuity and removes slowly changing trends from the data representation. Moreover, the block band matrices allow for convenient storage, which is very