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An Improved Hybrid Adjoint Method in External Aerodynamics Using Variational Technique for the Boundary Integral Based Optimal Objective Function Gradient

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Abstract. An improved hybrid adjoint method to the viscous, compressible Reynold-Averaged Navier-Stokes Equation (RANS) is developed for the computation of objective function gradient and demonstrated for external aerodynamic design optimization. In this paper, the main idea is to extend the previous coupling of the discrete and continuous adjoint method by the grid-node coordinates variation technique for the computation of the variation in the gradients of flow variables. This approach in combination with the Jacobian matrices of flow fluxes refrained the objective function from field integrals and coordinate transformation matrix. Thus, it opens up the possibility of employing the hybrid adjoint method to evaluate the subsequent objective function gradient analogous to many shape parameters, comprises of only boundary integrals. This avoids the grid regeneration in the geometry for every surface perturbation in a structured and unstructured grid. Hence, this viable technique reduces the overall CPU cost. Moreover, the new hybrid adjoint method has been successfully applied for the computation of accurate sensitivity derivatives. Finally, for the investigation of the presented numerical method, simulations are carried out on NACA0012 airfoil in a transonic regime and its accuracy and effectiveness related to the new gradient equation has been verified with the Finite Difference Method (FDM). The analysis reveals that the presented methodology for the optimization provides the designer with an indispensable CPU-cost effective tool to reshape the complex geometry airfoil surfaces, useful relative to the state-of-the-art; in a less computing time.

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1. Introduction

The goal of all-aerodynamics design methods is to modify the geometry complying with constraints to an appropriate shape, by the aerodynamics figure of merit. In this process, engineers incessantly plod to enhance their design applications in terms of operational efficacy and market appeal. At times, comparatively trivial and minor modifications in the complex design of the engineering system can lead to revelatory results. Specifically, design procedures based on CFD combined with gradient-based optimization techniques, have practically made it usable to eradicate tribulations in the decisive manufacturing procedure faced by the aerodynamics analysts. On account of the efficient and economical gradient evaluation, scientists have diverted their scrutiny to the control theory-based adjoint method [1, 2]. In general, considering the scope of exact sensitivities and cost regardless of the number of design variables, the adjoint method is a research hotspot in computational finance and shape optimization fields.

Present literature divulges the two categories of the adjoint method: the continuous and the discrete method. In the continuous method, adjoint equations are first linearized and then discretized by the governing Partial Differential Equation (PDE). As the applicability of the continuous method is initiated by Pirnonneau [3] for fluid dynamics, elliptic design problems. Whereby Jameson [4, 5] along with Reuther [6] extended this work, from potential flow to transonic and supersonic external flows governed by Euler and Navier Stokes Equation (NSE) for aeronautical designs. Later on, the research has been extended particularly for the internal flow turbomachinery design problems [7–11]. In the 1990s, contrary to the continuous method, the discrete method [12] has also attained the consideration of researchers. For the discrete method, first nonlinear discrete equations are linearized either manually or by Automatic Differentiation (AD) [13] software. While the implementation of AD could be possible via source code transformation TAPENADE [14, 15] or operator overloading ADOL-C tool [16]. And then discrete adjoint equations are obtained by transposing the linear operator. In this regard, a series of papers addressing the discrete sensitivity analysis of turbomachinery cascades [17–19] for constrained optimization and aerothermal optimization [20] have been published.

Meanwhile, Nadrajah and Jameson [21,22] have deeply explained the study of both continuous and discrete adjoint approaches by comparing their gradients, calculation speed, and accuracy. They concluded that the difference between them is small usually in shape optimization and finally has no significant effect on the results. But, still, both methods have their opportunities and obstacles. In the continuous method, lengthy mathematical development must be done but is better connected to the underlying physics. This approach allows the flexible discretization of the derived PDEs, which reduces the CPU cost but gives a semi-exact gradient. Despite this fact, the construction of the discrete adjoint equations and the boundary conditions is a clear and straight forward process. Another advantage is that because the equations are discretely adjoint to the flow equations the derivatives obtained are consistent with finite-difference gradients independent of the mesh size. The discrete adjoint gives an exact gradient with the