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CORNER-CUTTING SUBDIVISION SURFACES OF GENERAL DEGREES WITH PARAMETERS*

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Abstract

As a corner-cutting subdivision scheme, Lane-Riesefeld algorithm possesses the concise and unified form for generating uniform B-spline curves: vertex splitting plus repeated midpoint averaging. In this paper, we modify the second midpoint averaging step of the Lane-Riesefeld algorithm by introducing a parameter which controls the size of corner cutting, and generalize the strategy to arbitrary topological surfaces of general degree. By adjusting the free parameter, the proposed method can generate subdivision surfaces with flexible shapes. Experimental results demonstrate that our algorithm can produce subdivision surfaces with comparable or even better quality than the other state-of-the-art approaches by carefully choosing the free parameters.

Mathematics subject classification: 65D17, 68U07.
Key words: Lane-Riesenfeld algorithm, Spline curves, Subdivision curves/surfaces, Corner-cutting subdivision surfaces.

1. Introduction

Subdivision method aims to generate curves or surfaces with certain smoothness by repeatedly refining initial control polygons or meshes. Since it is simple and easy to implement, subdivision has become an important modeling method in computer graphics, computer aided geometric design and industrial modeling design.

Among the existing subdivision methods, many of them can be geometrically viewed as repetitive corner-cutting processes (i.e., corner-cutting subdivision). The earliest univariate subdivision scheme can be dated to the algorithm of producing uniform quadratic B-spline curves proposed in [1], which used the arithmetic averaging rule to gradually cut corners of the input control polygon. Lane and Riesenfeld [2] generalized this process to produce uniform B-spline curves of arbitrary degree. Similarly in the case of tensor product B-spline surfaces, Zorin et al. [3] proposed an uniform subdivision framework for arbitrary degree B-spline surfaces by replacing the arithmetic averaging with barycentric averaging, this framework can also be applied to quadrilateral meshes with arbitrary topology. In addition, the Catmull-Clark scheme [4] and Doo-Sabin scheme [5], which are the two most classic methods in the field of subdivision, also belong to the corner-cutting subdivision type. The only difference between these two methods and the one proposed in [3] is that both Catmull-Clark and Doo-Sabin algorithm use weighted averaging rather than barycentric averaging to improve the smoothness of their limit surfaces at extraordinary vertices or faces.

Due to the important role of corner-cutting schemes in subdivision algorithms, many researchers have studied their properties and proposed some new schemes of corner cutting, while

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using them to construct new subdivision algorithms. Aumann [6] analyzed the properties of linear corner-cutting subdivision curves. Noakes [7] replaced straight lines in [2] with geodesic segments to derive a nonlinear corner-cutting subdivision rule. The authors in [8] developed a new corner-cutting scheme, which provides a tension parameter to control the shape of subdivision surface and creates better handles and holes than the well-known Catmull-Clark scheme when applied to non-manifold meshes. Claes et al. [9] proposed a hexagonal scheme in combination with corner-cutting subdivision. Siddiqi and Rehan [10] introduced a 3-point and arity-2 corner-cutting scheme which generates a C^1-continuous limit curve, and the limit curve is closer to the initial control polygon than Chaikin's curve [1]. They further proposed a 3-point and arity-3 corner-cutting scheme whose limit curve possess better geometric properties [11]. The authors in [12] combined corner-cutting schemes and the four-point interpolation subdivision scheme in [13] to design a new algorithm called four point interpolatory-corner cutting subdivision for generating curves that interpolate some given vertices and approximate the other vertices.

As a fundamental subdivision algorithm for uniform B-spline curves, the Lane-Riesenfeld algorithm includes one vertex splitting plus several midpoint averagings in each subdivision step, and it can be made some modifications to deduce other meaningful subdivision schemes. For example, Schaefer et al. [14] created the subdivision schemes for functions of different types by replacing the midpoint averaging step in Lane-Riesenfeld algorithm with some simple nonlinear averaging operations such as geometric average, harmonic average, p-average and so on. Morin et al. [15] changed the midpoint averaging in Lane-Riesenfeld algorithm into weighted averaging to create a non-stationary subdivision scheme. We observe that it's the second midpoint averaging in this algorithm that contributes to cutting corners of the control polygon. Motivated by this observation, we change the second midpoint averaging into linear weighted averaging to adjust the size of the corner-cutting, and apply this idea to modify the approach in [3] for constructing corner-cutting subdivision surfaces of general degree with flexible shapes. It's worth mentioning that, like the schemes in [24] and [25], the degree of surface in our method can be general. In addition, when the degree of the subdivision surface is three, the corresponding limiting surfaces generated by our algorithm are equivalent to the Catmull-Clark subdivision surfaces with parameters introduced in the work [16], which can be applied in isogeometric analysis (similar to the works [26, 27]).

The continuity analysis of subdivision schemes has always been a challenging research problem. For the subdivision schemes that generalize B-splines directly, their smoothness can be analyzed by the properties of B-splines. For general corner-cutting subdivision algorithms, de Boor gave a convergence proof in [17] about corner-cutting schemes, which only utilized the fact that the corner-cutting operation is a convex transformation. Gregory and Qu [18] provided a sufficient condition for the proposition that the 2-point and arity-2 corner-cutting scheme is C^1-continuous, and de Boor added that the condition is true only when the corner-cutting scheme is local [19]. Later the authors in [20] gave a sufficient and necessary condition for the C^1-continuity of local corner-cutting schemes, and proved this condition from a geometrical perspective. In this paper, the continuity of the constructed subdivision surfaces is analyzed based on the relative conclusions in [21], and the range of the free parameter introduced in our method can be derived based on the continuity of the subdivision surfaces.

The remainder of the paper is organized as follows. Section 2 firstly revisit the corner-cutting subdivision scheme using barycenter averaging, then expound the essence of corner cutting and introduce our new surface subdivision method. In Section 3, we give the theoretical