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Direct Result for a Summation-Integral Type Modification of Szász–Mirakjan Operators

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Abstract. In this paper, study of direct result for a summation-integral type modification of Szász–Mirakjan operators is carried out. Calculation of moments, density result and a Voronvskaja-type result are also obtained.

Key Words: Szász–Mirakjan operators, K-functional, modulus of smoothness, Voronovskaja-type result.

AMS Subject Classifications: 41A10, 41A25, 41A36, 40A30

1 Introduction

In 1941, G. M. Mirakjan [9] defined the operators $SM_n : C_2[0,\infty) \to C[0,\infty)$ for any $x \in [0,\infty)$ and for any $n \in \mathbb{N}$ given by,

$$SM_n(f;x) = \sum_{k=0}^{\infty} s_{n,k}(x) f\left(\frac{k}{n}\right), \tag{1.1}$$

where

$$s_{n,k}(x) = e^{-nx}\frac{(nx)^k}{k!}, \qquad 0 \le x < \infty. \tag{1.2}$$

and

$$C_2[0,\infty) = \left\{ f \in C[0,\infty) : \lim_{x\to\infty} \frac{f(x)}{1+x^2} \text{ exists and is finite} \right\}.$$

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The operators $(SM_n)_{n\in\mathbb{N}}$ are named Szász-Mirakjan operators, where $s_{n,k}$'s are Szász basis functions. They were extensively studied in 1950 by O. Szász [15].

Durrmeyer [3] defined the summation-integral type approximation process, using the Bernstein polynomials, as

$$D_n(f;x)=(n+1)\sum_{k=0}^{n}b_{n,k}(x)\left(\int_0^1 b_{n,k}(t)f(t)dt\right), \tag{1.3}$$

where Bernstein polynomial are given by

$$B_n(f;x)=\sum_{k=0}^{n}b_{n,k}(x)f\left(\frac{k}{n}\right),$$

and

$$b_{n,k}(x)=\left(\begin{array}{c} n \\ k \end{array}\right)x^k(1-x)^{n-k},$$

$0\leq x\leq 1$, $k=0,1,\cdots,n$ and $n\in\mathbb{N}$.

Derriennic [2] studied the operators given by (1.3) extensively. Motivated by Derriennic, Sahai and Prasad [14] studied many properties of the modified Lupaş operators of the type

$$M_n(f;x)=(n-1)\sum_{k=0}^{\infty}p_{n,k}(x)\left(\int_0^\infty p_{n,k}(t)f(t)dt\right), \tag{1.4}$$

where

$$p_{n,k}(x)=\left(\begin{array}{c} n+k-1 \\ k \end{array}\right)\frac{x^k}{(1+x)^{n+k}},$$

$0\leq x<\infty$, $k=0,1,2,\cdots$ and $n\in\mathbb{N}$.

Mazhar and Totik [8] introduced two Durrmeyer type modifications of Szász-Mirakjan operators (1.1) as

$$\bar{S}_n(f;x)=f(0)s_{n,0}(x)+n\sum_{k=1}^{\infty}s_{n,k}(x)\int_0^\infty s_{n,k-1}(t)f(t)dt \tag{1.5}$$

and

$$S_n(f;x)=n\sum_{k=0}^{\infty}s_{n,k}(x)\int_0^\infty s_{n,k}(t)f(t)dt, \tag{1.6}$$

where $s_{n,k}$'s are as given by (1.2).

Various properties, like global approximation in weight spaces, uniform approximation, simultaneous approximation, weighted approximations, of these operators, their generalizations and modifications are studied over the years. We can mention some important studies of this type (see [4–7, 10–12]).