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A Generalization of a Troubled-Cell Indicator to h-Adaptive Meshes for Discontinuous Galerkin Methods

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Abstract. We generalize the troubled-cell indicator on unstructured triangular meshes recently introduced by Fu and Shu (J. Comput. Phys., 347 (2017), pp. 305–327) to h-adaptive rectangular meshes where hanging nodes exist. The generalized troubled-cell indicator keeps the good properties of simplicity, compactness and insensitivity to particular test cases. Numerical tests on the two-dimensional scalar Burgers' equation and hyperbolic systems of Euler equations demonstrate the good performance of the generalized indicator. The results on both uniform and h-adaptive meshes indicate that the generalized indicator is able to capture shocks effectively without any PDE-sensitive parameter to tune.

AMS subject classifications: 65M60, 35L65

Key words: Troubled-cell indicator, discontinuous Galerkin method, adaptive mesh, conservation law.

1 Introduction

In this paper, we generalize the recently proposed troubled-cell indicator in [12] to two-dimensional h-adaptive meshes for the discontinuous Galerkin (DG) methods for solving hyperbolic conservation laws. This generalization is nontrivial due to the appearance of hanging nodes in the mesh.

The DG method was pioneered by Reed and Hill [24] in the study of neutron transport equations in 1973. A major development of this method was carried out by Cockburn et al. in a series of papers [3–6], in which a framework to solve nonlinear time-dependent hyperbolic conservation laws was established. They adopted explicit, nonlinearly stable high order Runge-Kutta (RK) time discretizations [28], DG space discretizations with exact or approximate Riemann solvers as interface fluxes and a total variation bounded

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(TVB) nonlinear limiter [27] to achieve nonoscillatory properties. These schemes are termed RKDG methods. Completely discontinuous piecewise polynomial space was employed for the numerical solutions and the test functions, which made the scheme highly parallelizable and easy to handle complicated geometries, boundary conditions and hp adaptation. There have been massive works on DG methods in diverse areas in the literature, for example, the ones in [18, 19, 31].

The adaptive method is an important research direction of the DG method, which is widely used to increase spatial and temporal resolution of numerical simulations beyond the limits imposed by the available hardware and to save the computational cost. There are a lot of adaptive DG schemes in the literature, especially the h-adaptive schemes which are based on local mesh refinement and coarsening. We refer, for instance, to Flaherty et al. [1, 9, 11, 25], Hartmann and Houston [15], Dedner et al. [8], Tian et al. [30] and Zhu et al. [37–42].

Solutions of nonlinear hyperbolic conservation laws usually develop discontinuities even though the initial conditions are sufficiently smooth, which leads to great difficulty in numerical simulation. An important component of the RKDG method for solving conservation laws with strong shocks is a nonlinear limiter. It is used to detect discontinuities and control spurious oscillations near such discontinuities. According to the framework of limiters proposed by Qiu and Shu [22], a limiter is typically composed of two parts. The first part detects the troubled cells (those cells near the discontinuities where solutions need to be limited for stability) in the discontinuous regions and is called a troubled-cell indicator. The second part is a solution reconstruction method, which reconstructs the numerical solution in the detected cells to control the spurious oscillations.

In [21], various troubled-cell indicators, coupled with a weighted essentially non-oscillatory (WENO) solution reconstruction method [22], were systematically investigated and compared for the RKDG method. Most of these indicators came from limiters, including the minmod type TVB limiter [3–6], the moment based limiters [1, 2], the monotonicity-preserving (MP) limiter [29] and the modified MP limiter [26]. The other two indicators were Harten's subcell resolution idea [14] and the shock detector of Krivodonova et al. (KXRCF) [16]. In the comparison, no universally better indicator for every problem was found. Troubled-cell indicators continued to receive attention in recent years. Persson and Peraire [20] presented a shock capturing strategy for higher order DG approximations. Dumbser et al. [10] introduced a novel a posteriori subcell limiter which contains a new troubled-cell indicator. Vuik and Ryan [32] proposed a multiwavelet troubled-cell indicator. In their another work [33], outlier detection was used to automatically determine the tuning parameter for various troubled-cell indicators. A modification of this work was given by Gao et al. [13]. In 2017, Fu and Shu [12] introduced a new indicator without PDE sensitive parameters to tune. In 2018, Ray and Hesthaven [23] proposed a universal troubled-cell indicator using artificial neural networks which was also independent of problem-dependent parameters.

Usually, troubled-cell indicators are based on certain local data, typically the data from the target cell and its immediate neighbors so that the indicator can be as compact