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A BLOW-UP RESULT FOR A CLASS DOUBLY NONLINEAR PARABOLIC EQUATIONS WITH VARIABLE-EXPONENT NONLINEARITIES[*†]

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Abstract

This paper deals with the following doubly nonlinear parabolic equations $(u+|u|^{r(x)-2}u)_t-\text{div}(|\nabla u|^{m(x)-2}\nabla u)=|u|^{p(x)-2}u$, where the exponents of nonlinearity $r(x), m(x)$ and $p(x)$ are given functions. Under some appropriate assumptions on the exponents of nonlinearity, and with certain initial data, a blow-up result is established with positive initial energy.

Keywords doubly nonlinear parabolic equations; variable-exponent nonlinearities; blow-up

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1 Introduction

Let Ω be a bounded domain in R^n with a smooth boundary $\partial\Omega$. We consider the following initial boundary value problem

$$(u+|u|^{r(x)-2}u)_t-\text{div}(|\nabla u|^{m(x)-2}\nabla u)=|u|^{p(x)-2}u, \tag{1.1}$$

$$u=0, \quad x\in\partial\Omega, \tag{1.2}$$

$$u(x,0)=u_0(x), \quad x\in\Omega, \tag{1.3}$$

where the exponents $m(\cdot)$, $p(\cdot)$ and $r(\cdot)$ are given measurable functions on Ω satisfying

$$2\le r^-\le r(x)\le r^+<p^-\le p(x)\le p^+<m^*(x), \tag{1.4}$$

$$2\le m^-\le m(x)\le m^+<p^-\le p(x)\le p^+<m^*(x) \tag{1.5}$$

with

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$$r^- = \operatorname*{ess\,inf}_{x\in\Omega} r(x), \quad r^+ = \operatorname*{ess\,sup}_{x\in\Omega} r(x), \quad p^- = \operatorname*{ess\,inf}_{x\in\Omega} p(x),$$
$$p^+ = \operatorname*{ess\,sup}_{x\in\Omega} p(x), \quad m^- = \operatorname*{ess\,inf}_{x\in\Omega} m(x), \quad m^+ = \operatorname*{ess\,sup}_{x\in\Omega} m(x).$$

The exponent function $m(x)$ satisfies

$$m^*(x) = \begin{cases} \dfrac{nm(x)}{\operatorname*{ess\,sup}_{x\in\Omega}(n-m(x))}, & \text{if } m^+ < n, \\ \infty, & \text{if } m^+ \geq n, \end{cases} \tag{1.6}$$

and the exponent functions $m(x)$, $p(x)$, and $r(x)$ also satisfy the log-Holder continuity condition

$$|q(x)-q(y)| \leq -\frac{A}{\log|x-y|}, \quad \text{for almost everywhere } x, y \in \Omega, \text{ with } |x-y| < \delta, \tag{1.7}$$

where $A \geq 0$, $0 < \delta < 1$. The term $\Delta_{m(\cdot)}u = \operatorname{div}(|\nabla u|^{m(x)-2}\nabla u)$ is called $m(\cdot)$-Laplacian.

Problem (1.1)-(1.3) has been widely used in many mathematical models of various applied sciences such as thin liquid films, flows of electro-rheological fluids etc. The interested readers may refer to [1-3] and the further references therein.

In case of constant-exponent nonlinearities (that is the exponents $m(\cdot)$, $p(\cdot)$ and $r(\cdot)$ are constants), the local, global existence, nonexistence and long-time behavior for problem (1.1)-(1.3) have been established by many authors, see, e.g., [4-10] and the further references therein.

In recent years, the research on parabolic equations, elliptic equations and hyperbolic equations with nonlinearities of variable-exponent has been an interesting topic. For more results, see the recent papers [11-13]. For problem (1.1)-(1.3), the authors in [1-3] applied Galerkin method to establish the existence of a solutions. But there are seldom works on blow-up of solutions. In this paper, we intend to find sufficient conditions on $m(\cdot), p(\cdot), r(\cdot)$, and the initial data for which the blowup occurs. Our result extends that established in [5] from constant-exponent nonlinearities to variable-exponent nonlinearities.

The present work is organized as follows. In Section 2 we present some notations and materials needed for our work while in Section 3 we are devoted to proving our main result.

2 Preliminaries

In this section, we present some notations and materials needed for our work. Firstly, we give some preliminary facts about Lebesgue and Sobolev spaces with