JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **34**, No. 1, pp. 94-102

doi: 10.4208/jpde.v34.n1.7
March 2021

Lower Bound Estimate of Blow Up Time for the Porous Medium Equations under Dirichlet and Neumann Boundary Conditions

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Received 1 August 2020; Accepted 31 October 2020

Abstract. In this paper, we establish the lower bounds estimate of the blow up time for solutions to the nonlocal cross-coupled porous medium equations with nonlocal source terms under Dirichlet and Neumann boundary conditions. The results are obtained by using some differential inequality technique.

AMS Subject Classifications: 35K20, 35K55, 35K60, 35K65

Chinese Library Classifications: O175.26

Key Words: Lower bounds; Blow up time; Nonlocal source terms; Dirichlet and Neumann boundary conditions.

1 Introduction

In this article, we consider the lower bound of blow up time for solutions of the nonlocal cross-coupled porous medium equations

$$u_t=\Delta u^m+\int v^p \mathrm{d}x, \qquad (x,t)\in\Omega\times(0,t^\star), \tag{1.1}$$

$$v_t=\Delta v^n+\int u^q \mathrm{d}x, \qquad (x,t)\in\Omega\times(0,t^\star), \tag{1.2}$$

and continuous bounded initial values

$$u(x,0)=u_0(x), \quad v(x,0)=v_0(x), \quad x\in\Omega, \tag{1.3}$$

under Dirichlet boundary condition

$$u(x,t)=v(x,t)=0, \qquad (x,t)\in\partial\Omega\times(0,t^\star), \tag{1.4}$$

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or Neumann boundary condition

$$\frac{\partial u^m}{\partial v} = lu, \quad \frac{\partial v^n}{\partial v} = lv, \qquad (x,t) \in \partial\Omega \times (0,t^\star), \tag{1.5}$$

where $\Omega \in R^3$ is a bounded region of $\partial\Omega$ with a smooth boundary, and satisfies that $p > m > 1, q > n > 1$, v is the unit external normal vector in the external normal direction of $\partial\Omega$. There are many research achievements on the lower bound estimation of blow up time for the solution of a single porous media equation, see, e.g., [1-3]. Liu, et al. [1] studied the following nonlocal porous equation with Dirichlet boundary conditions

$$u_t = \Delta u^m + u^p \int u^q \mathrm{d}x, \qquad (x,t) \in \Omega \times (0,t^\star), \tag{1.6}$$

They have obtained the lower bound of the blow up time of the solution which

$$t^\star \geq C_7 [\int_\Omega u_0^\alpha (p+q-1) \mathrm{d}x]^{-C_6}.$$

and homogeneous Neumann boundary conditions, the lower bound of the blow up time of the solution which

$$t^\star \geq \int_{\eta(0)}^{\infty} \frac{\mathrm{d}\xi}{K_5 \xi^{\frac{\alpha+1}{\alpha}} + K_6 \xi^{\frac{(\alpha+1)(p+q-1)}{\alpha(p+q-1)-(p+q-m)}}}.$$

Liu [2] considered Eq. (1.6) with Robin boundary conditions, they have obtained the lower bound of the blow up time of the solution which

$$t^\star \geq \int_{\phi(0)}^{\infty} \frac{\mathrm{d}\eta}{ms|\Omega| K_4 \eta^{\frac{ms+s}{ms+m-1}} + ms|\Omega| K_5 \eta^{\frac{ms+s}{n(m-1)(s+1)}}}.$$

Fang and Chai [3] studied Eq. (1.6) with Neumann boundary conditions

$$\frac{\partial u^m}{\partial v} = lu, \qquad (x,t) \in \partial\Omega \times (0,t^\star),$$

the lower bound of the blow up time of the solution which

$$t^\star \geq \int_{\eta(0)}^{\eta(t)} \frac{\mathrm{d}\xi}{C_1 \xi + C_2 \xi^{\frac{\alpha(p+q-1)-(m-1)}{\alpha(p+q-1)}} + C_2 \xi^{\frac{(\alpha+1)}{\alpha}} + C_3 \xi^{\frac{(\alpha+1)(p+q-1)}{\alpha(p+q-1)-(p+q-m)}}},$$

when $l > 0$ of (1.5). The lower bound of the blow up time of the solution which

$$t^\star \geq \int_{\eta(0)}^{\eta(t)} \frac{\mathrm{d}\xi}{K_1 \xi^{\frac{(\alpha+1)}{\alpha}} + K_2 \xi^{\frac{(\alpha+1)(p+q-1)}{\alpha(p+q-1)-(p+q-m)}}},$$

when $l < 0$ of Eq. (1.5).