Algebraic Multigrid Block Preconditioning for Multi-Group Radiation Diffusion Equations

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Abstract. The paper focuses on developing and studying efficient block preconditioners based on classical algebraic multigrid (AMG) for the large-scale sparse linear systems arising from the fully coupled and implicitly cell-centered finite volume discretization of multi-group radiation diffusion equations, whose coefficient matrices can be rearranged into the $(G+2) \times (G+2)$ block form, where G is the number of energy groups. The preconditioning techniques are the monolithic classical AMG method, physical-variable based coarsening two-level algorithm and two types of block Schur complement preconditioners. The classical AMG method is applied to solve the subsystems which originate in the last three block preconditioners. The coupling strength and diagonal dominance are further explored to improve performance. We take advantage of representative one- and twenty-group linear systems from capsule implosion simulations to test the robustness, efficiency, strong and weak parallel scaling properties of the proposed methods. Numerical results demonstrate that block preconditioners lead to mesh- and problem-independent convergence, outperform the frequently-used AMG preconditioner and scale well both algorithmically and in parallel.

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1   Introduction

The multi-group radiation diffusion (MGD) equations have a broad range applications, including inertial confinement fusion (ICF) and astrophysics [31]. As a result of the complicated nonlinear couplings among dozens of physical quantities at multiple temporal and spatial scales, MGD equations are often discretized by the finite volume method allowing for local conservations [16, 21–23, 33, 35, 42, 43, 52], resulting in a series of non-symmetric but positive definite large-scale sparse linear systems. The overall complexity increases not only with mesh sizes but also with the level of couplings between physical quantities. It must be emphasized that these numerical solutions play a time-consuming role (about 80% in general) in ICF numerical simulations, due to the fact that the coefficient matrices are invariably ill-conditioned.

To effectively address the aforesaid bottlenecks, numerous preconditioned Krylov subspace methods have been proposed in an efficient and scalable manner over the past decades, see [5, 26, 32, 55–58, 60–62] and references cited therein. These preconditioners are conceived as approximate inverses and they fall into the category of incomplete LU factorizations, domain decomposition preconditioners, monolithic algebraic multigrid (AMG) methods and their symmetric / nonsymmetric combinations. Since each of those coefficient matrices has an underlying block structure, one can also determine a block preconditioner to separate the global problem into easier-to-solve subproblems and form an object-oriented framework to allow for code-reuse and incorporate single-physics experience into multi-physics simulations. The objective is to approximately invert numerous individual scalar systems instead of the fully coupled systems. Preconditioners of this type had been proposed and analyzed in the literature, including the block diagonal preconditioner [17, 29, 65], block lower / upper triangular preconditioner [3, 7, 12], product (splitting) preconditioner [38, 48, 64] and constraint preconditioner [4, 18, 28]. Block preconditioners with multigrid components had proven very successful in a variety of applications, e.g., liquid crystal directors modeling [6], multiphase flow in porous media [8], mixed Stokes / Darcy model [11], coupled poromechanics [13], linear elasticity in mixed form [14], Stokes problem [15], incompressible Navier-Stokes problem [19,49], Biot and multiple-network flux-based poroelasticity models [25], field-scale reservoir simulation [27], incompressible magnetohydrodynamics model [30], Dirichlet biharmonic problem [36], electrical activity in the heart [46], Brinkman problem [47], all-speed melt pool flow physics [50] and fully coupled flow and geomechanics [51]. Our focus in this work is on the block preconditioning based on the classical AMG method owing to its general applicability, high efficiency and easy-to-use user interface.

In the recent work [1], four types of operator-based preconditioners have been developed in the Jacobian-free Newton-Krylov method for solving two-dimensional three-temperature energy equations. These preconditioners are application specific, relying on physical properties of the energy equations as well as different linearizations on different terms in the nonlinear residual. They are demonstrated numerically to be very effective. However, the corresponding preconditioning matrix has to be assembled explicitly in