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EXISTENCE OF UNBOUNDED SOLUTIONS FOR A n-TH ORDER BVPS WITH A p-LAPLACIAN[*†]

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Abstract

This paper considers the solvability of boundary value problems with a p-Laplacian

$$\begin{cases} -(\phi_p(u^{(n-1)}(t)))' = q(t)f(t, u(t), \cdots, u^{(n-1)}(t)), \quad 0 < t < +\infty, \\ u^{(i)}(0) = A_i, \quad i = 0, 1, \cdots, n-3, \\ u^{(n-2)}(0) - au^{(n-1)}(0) = B, \quad u^{(n-1)}(+\infty) = C. \end{cases}$$

By using the methods of upper and lower solution, the schäuder fixed point theorem, and the degree theory, we obtain the existence of one and triple solutions. This paper generalizes several problems due to the dependence on the p-Laplacian operator, the $n-1$-th derivative not only in the differential equation but also in the boundary conditions. The most interesting point is that the solutions may be unbounded.

Keywords p-Laplacian; upper solutions; lower solutions; infinite interval; degree theory

2000 Mathematics Subject Classification 34B15; 34B40

1 Introduction

This paper discusses the n-th differential equation with a p-Laplacian operator on the infinite interval

$$-(\phi_p(u^{(n-1)}(t)))' = q(t)f(t, u(t), \cdots, u^{(n-1)}(t)), \quad 0 < t < +\infty \tag{1.1}$$

with the Sturm-Liouville boundary conditions:

$$\begin{cases} u^{(i)}(0) = A_i, \quad i = 0, 1, \cdots, n-3, \\ u^{(n-2)}(0) - au^{(n-1)}(0) = B, \\ u^{(n-1)}(+\infty) = C, \end{cases} \tag{1.2}$$

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where $\phi_p(s) = |s|^{p-2}s$, $p > 1$, $q : (0, +\infty) \to (0, +\infty)$, $f : [0, +\infty) \times \mathbb{R}^n \to \mathbb{R}$ are continuous, $a > 0$, $A_i, B, C \in \mathbb{R}$, $i = 0, 1, \cdots, n-3$, $u(+\infty) = \lim\limits_{t\to+\infty} u(t)$.

In recent years, the p-Laplacian boundary value problem attracted considerable attention due to their application in mechanics, astrophysics and frequent appearance in the classical electrical. The existence of the solutions, positive or multiple ones have been discussed by using the upper and lower solution method, the fixed point theory, the shooting method, the critical point theory. See [2,4,13-20] for high order boundary value problems. In [2], John R. Graef et al. considered the higher order ϕ-Laplacian BVP with the generalized Sturm-Liouville boundary condition

$$\begin{cases} u^{(i)}(0) = A_i, \quad i = 0, 1, \cdots, n-3, \\ u^{(n-2)}(0) - au^{(n-1)}(0) = B, \\ u^{(n-1)}(+\infty) = C. \end{cases}$$

[16,17] considered even order differential equations, [14,18] studied multi-point BVPs. For differential equations with Laplace operator, some scholars have explored them on unbounded domains, but just for first or second order (see [3-8]).

Inspired by the above works, we intend to use the upper and lower solution method to solve the higher order differential equation with p-Laplacian operator on the infinite interval. Here, the compactness of the infinite interval is worth attention. We assume problem (1.1) with (1.2) exists a pair of upper and lower solutions and the nonlinear function f satisfies a Nagumo condition. Through using the truncation method and upper and lower solution method, we give a priori boundary of the truncation problem. Then the Schäder fixed point theorems guarantee the existence of the solution of problem (1.1) with (1.2). On this basis, by assuming that there are two pairs of upper and lower solutions, the existence of multiple solutions is discussed. In the last part, two examples are included to illustrate the main results in this paper. This work is done to show how the method of upper and lower solution can be used to establish the existence of unbounded solutions of p-Laplacian model on an infinite interval.

2 Preliminaries

Consider a space X defined by

$$X = \left\{ u \in C^{n-1}[0, +\infty), \lim_{t\to+\infty} \frac{u^{(i)}(t)}{v_i(t)} \text{exist}, i = 0, 1, \cdots, n-1 \right\}$$

with the norm $\|\cdot\|$ given by

$$\|u\| = \max\{\|u\|_0, \|u\|_1, \cdots, \|u\|_{n-1}\},$$

where $v_i(t) = 1 + t^{n-1-i}$, and