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BIFURCATION ANALYSIS OF A CLASS OF PLANAR PIECEWISE SMOOTH LINEAR-QUADRATIC SYSTEM*†

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Abstract

In this paper, we consider a planar piecewise smooth differential system consisting of a linear system and a quadratic Hamiltonian system. The quadratic system has some folds on the discontinuity line. The linear system may have a focus, saddle or node. Our results show that this piecewise smooth differential system will have two limit cycles and a sliding cycle. Moreover, this piecewise smooth system will undergo pseudo-homoclinic bifurcation, Hopf bifurcation and critical crossing bifurcation CC. Some examples are given to illustrate our results.

Keywords piecewise smooth systems; limit cycle; sliding cycle; pseudo-homoclinic bifurcation; critical crossing bifurcation CC

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1 Introduction

The bifurcation theory of planar smooth differential systems has developed very fast since D. Hilbert put up the famous Hilbert's 16th problem (see e.g. [19] and [23]). In recent years, piecewise smooth (PWS for short) dynamical systems with some parameters have been widely applied in many fields, such as mechanics, electronics, control theory, biology, economy and so on. They are also used to explain some phenomena such as pest control or model some mechanical systems exhibiting dry

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friction and electrical circuits having switches and so on. These wide applications and study of Hilbert's 16th problem are important sources of motivation of bifurcation analysis in PWS systems (see e.g. [7,16,20,30-32]).

Filippov established some systematic methods to study qualitative theory of PWS systems in his book [9]. Kuznetsov et al. studied one-parameter bifurcations in planar Filippov systems. They gave an overview of all codimension one bifurcations including some novel bifurcation phenomena that can only appear in planar PWS systems in [21]. Guardia et al. studied local and global bifurcation in PWS systems in [14]. Many novel bifurcation phenomena that will not be seen in smooth systems have been hot issues of bifurcation problems of PWS systems such as sliding bifurcation, critical crossing cycle bifurcation and so on. Freire et al. studied critical crossing cycle bifurcation and pointed out that critical crossing cycle bifurcation CC can occur in co-dimension one bifurcation, but critical crossing cycle bifurcation CC_2 cannot occur in co-dimension one bifurcation problems (see [12]).

When subsystems of planar PWS systems have the same type of singularities and different types of singularities, their bifurcation problems have been widely studied in the past few years. Even for planar PWS linear systems defined in two zones, their bifurcation problems are not easy to be studied. People have found many novel bifurcation phenomena that will not appear in smooth linear systems (see [13, 15]). It is not easy for people to discuss bifurcation phenomena of planar piecewise linear systems with many parameters. Luckily, Freire et al. gave a Liénard-like canonical form for a class of piecewise linear systems with two zones. When each subsystem has no equilibrium point in its own zone and if each subsystem has a focus, they showed that two limit cycles can exist (see e.g. [10]). PWS linear systems with node-node dynamics and saddle-saddle types were considered in [17] and [18], respectively.

Recently, bifurcation phenomena of planar PWS systems that are constituted by linear system and quadratic Hamiltonian system have been studied by some authors (see e.g. [22,27-29]).

Li and Huang [22] considered the following PWS system

$$(\dot{u}, \dot{v}) = \begin{cases} (a_0 + \eta, b_1^+ u + b_2^+ u^2 + \epsilon^+), & \text{if } v > 0, \\ (-a_0 + a_1^- u + a_2^- v - \eta, b_1^- u + \epsilon^-), & \text{if } v < 0. \end{cases} \tag{1}$$

Under their assumptions, the linear system has a saddle. They got the following system which is topologically equivalent to system (1)

$$(\dot{x}, \dot{y}) = \begin{cases} (1, -2x + 3lx^2 + \epsilon_1), & \text{if } y > 0, \\ (-1 + mx + ny, -x + \epsilon_2), & \text{if } y < 0. \end{cases} \tag{2}$$

The unperturbed system of (2) is the following system (3). See equation (7) in [22].