Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2020-0137

Vol. **30**, No. 1, pp. 227-254
July 2021

A Simple 3D Immersed Interface Method for Stokes Flow with Singular Forces on Staggered Grids

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Received 15 July 2020; Accepted (in revised version) 8 December 2020

Abstract. In this paper, a fairly simple 3D immersed interface method based on the CG-Uzawa type method and the level set representation of the interface is employed for solving three-dimensional Stokes flow with singular forces along the interface. The method is to apply the Taylor's expansions only along the normal direction and incorporate the jump conditions up to the second normal derivatives into the finite difference schemes. A second order geometric iteration algorithm is employed for computing orthogonal projections on the surface with third-order accuracy. The Stokes equations are discretized involving the correction terms on staggered grids and then solved by the conjugate gradient Uzawa type method. The major advantages of the present method are the special simplicity, the ability in handling the Dirichlet boundary conditions, and no need of the pressure boundary condition. The method can also preserve the volume conservation and the discrete divergence free condition very well. The numerical results show that the proposed method is second order accurate and efficient.

AMS subject classifications: 65M06, 76D07

Key words: 3D immersed interface method, CG-Uzawa method, Stokes flow, level set method, staggered grids, singular forces.

1 Introduction

Numerical simulations for the interface problems have become one of the hot topics in computational fluid mechanics because these problems have important practical implications in many applications in physics and biology. The challenges for flow problems

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with moving interfaces are how to track and evolve the motion of the interface accurately, how to conserve the volume of the enclosed interface well, and how to preserve the discrete divergence free constraint well. This paper will focus on three-dimensional incompressible viscous Stokes flows in a Cartesian grid. C.Peskin proposed an immersed boundary method (IB-Method) which is originally aimed at the fluid dynamics of blood flow in [24,25]. This method converts the complex structure of the interface into the force source terms in the momentum equations in Cartesian grid and applies these forces to grid points near the interface. More precisely, they track the interface with the front-tracking method which has better volume conservation than level-set method, but there are still some problems in three-dimensional case. In the past few decades the IB-Method was widely applied to many other problems such as biofilm processes and biomechanics, see [3, 11, 23, 26] for a detail. And there are some other Cartesian grid methods such as [1,2,4,5,7,15,35] which have been proposed for the interface problem. This series of approaches involve the methods such as spectral method, finite volume method and finite element method. LeVeque employed the implicit immersed boundary method to simulate viscous incompressible flows with immerse elastic membranes for three-dimensional problem [14] in 2009. IB-Method is very efficient but it doesn't achieve global second order spatial accuracy in general. About the analysis for convergence, the interested readers are referred to the references [42, 43].

Alternative method to solve the interface problem is immersed interface method (IIM) which was originally proposed for elliptic interface problem by Li and LeVeque [16]. A standard five-point central finite difference scheme is used at the points away from the interface in IIM. For the points near the interface the correction terms computed from the jump conditions are incorporated to the difference scheme. So that the accuracy can be guaranteed in the whole domain. The most significant advantage of the IIM is that it can achieve global second order accuracy [19, 34]. Then the IIM was applied to Stokes and Navier-Stokes flows in [17, 22]. In [18], a fast iterative IIM for elliptic interface problem with discontinuous coefficient was proposed. The core idea of this fast algorithm is to introduce an augmented variable. The GMRES iterative method was adopted to compute the augmented variable. Later on, the fast algorithm was extended to Stokes equations with continuous viscosity in [33] and discontinuous viscosity in [21]. A detail overview of IIM can be found in [20]. In recent years, a coupled immersed interface and level set method for three-dimensional steady Stokes equations has been proposed in [37]. In this method the Stokes equations were divided into two Poisson systems, one for the velocity field and the other for the pressure field. As discussed in [27], the boundary conditions for pressure should be considered. For testing purpose the Dirichlet and Neumann boundary conditions have been used. But there are some additional conditions for pressure.

Recently, Lai proposed a simple version of IIM [13] for Stokes equations in 2D, which applies the Taylor expansion only along the normal direction and then incorporates the correction terms into the difference scheme. Fewer jump conditions are required when computing the correction terms compared to the original IIM, which leads to fairly sim-