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Notes on Automorphisms of Prime Rings

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Abstract: Let R be a prime ring, L a noncentral Lie ideal and σ a nontrivial automorphism of R such that $u^s\sigma(u)u^t = 0$ for all $u \in L$, where s, t are fixed non-negative integers. If either $\mathrm{char}R > s+t$ or $\mathrm{char}R = 0$, then R satisfies s_4, the standard identity in four variables. We also examine the identity $(\sigma([x, y]) - [x, y])^n = 0$ for all $x, y \in I$, where I is a nonzero ideal of R and n is a fixed positive integer. If either $\mathrm{char}R > n$ or $\mathrm{char}R = 0$, then R is commutative.

Key words: prime ring, Lie ideal, automorphism

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1 Introduction

The standard identity s_4 in four variables is defined as follows:

$$s_4 = \sum_{\tau}(-1)^{\tau} X_{\tau(1)}X_{\tau(2)}X_{\tau(3)}X_{\tau(4)},$$

where $(-1)^{\tau}$ is the sign of the permutation τ of the symmetric group of degree 4. In the following, unless stated otherwise, R always denotes a prime ring with its center $Z(R)$ and Martindale quotient ring Q. The center of Q, denoted by C, is called the extended centroid of R (we refer the reader to [1] for these terminologies). For any x, $y \in R$, the symbol $[x, y]$ stands for the commutator $xy - yx$. An additive subgroup U of R is said to be a Lie ideal of R if $[u, r] \in U$ for all $u \in U$ and $r \in R$. For nonempty subsets A, B of R, let $[A, B]$ be the additive subgroup generated by all the elements of the form $[a, b]$ with $a \in A$ and $b \in B$. Recall that a ring R is prime if for any a, $b \in R$, $aRb = (0)$ implies $a = 0$ or $b = 0$, and is semiprime if for any $a \in R$, $aRa = (0)$ implies $a = 0$. An additive mapping $d : R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$. Starting from this

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definition, Brešar[2] first introduced the definition of generalized derivations: An additive mapping $F : R \to R$ is called a generalized derivation if there exists a derivation $d : R \to R$ such that $F(xy) = F(x)y + xd(y)$ holds for all $x, y \in R$, and d is called the associated derivation of F. Hence, the concept of generalized derivations covers the concepts of both derivations and left multipliers (i.e., the additive mappings satisfying $F(xy) = F(x)y$ for all $x, y \in R$). Basic examples are derivations and generalized inner derivations (i.e., mappings of the type $x \mapsto ax + xb$ for some $a, b \in R$). We prefer to call such mappings generalized inner derivations for the reason that they present a generalization of the concept of inner derivations (i.e., mappings of the form $x \mapsto ax - xa$ for some $a \in R$).

This paper is included in a line of investigation concerning the relationship between the global structure of a ring R and the behaviors of some additive mappings defined on R that satisfy certain special identities. A well-known result of Herstein[3] states that if ρ is a right ideal of R such that $u^n = 0$ for all $u \in \rho$, where n is a fixed positive integer, then $\rho = 0$. Chang and Lin[4] considered the situation when $d(u)u^n = 0$ for all $u \in \rho$, where d is a nonzero derivation of R. Dhara and De Filippis[5] studied the case when $u^s H(u)u^t = 0$ for all $u \in L$, where L is a noncommutative Lie ideal of R, H is a generalized derivation of R and s, t are fixed non-negative integers. More precisely, they proved the following: Let R be a prime ring, H a nonzero generalized derivation of R and L a noncommutative Lie ideal of R. Suppose that $u^s H(u)u^t = 0$ for all $u \in L$. Then R satisfies s_4, the standard identity in four variables. On the other hand, Carini and De Filippis[6] proved that if R is a prime ring with $\text{char}R \neq 2$ and such that $[d(u), u]^n = 0$ for all $u \in L$, where L is a noncentral Lie ideal and d is a nonzero derivation of R, then R is commutative. Wang[7] also discussed the identity $[\sigma(u), u]^n = 0$ replacing the derivation d by an automorphism σ of R and obtained that R satisfies s_4. Motivated by the previous results, our first objective in this note is to study the identity $u^s\sigma(u)u^t = 0$ for all $u \in L$, where L is a noncentral Lie ideal and σ is an automorphism of the prime ring R, and then describe the structure of R.

During the past few decades, there has been an ongoing interest in concerning the relationship between the commutativity of a ring and the existence of certain specific types of derivations (see [8–9] for a partial bibliography, where further references can be found). The first result in this direction is due to Posner[10] who proved that a prime ring R admitting a nonzero derivation d such that $[d(x), x] \in Z(R)$ for all $x \in R$ must be commutative. This result was subsequently refined and extended by a number of authors, and we refer the readers to [11–13] for further references. In 1992, Daif and Bell[14] showed that if in a semiprime ring R there exist a nonzero ideal I of R and a derivation d such that $d([x, y]) = [x, y]$ for all $x, y \in I$, then $I \subseteq Z(R)$. If R is a prime ring, this implies that R is commutative. De Filippis[15] obtained the commutativity of prime rings when the derivation d is replaced by an automorphism σ. Later, Quadri *et al.*[16] extended Daif's result to generalized derivations. Ashraf and Ali[17] proved that R is commutative in the setting of left multipliers. In 2002, De Filippis[18] obtained the following result: Let R be a prime ring without non-zero nil right ideal, d a nonzero derivation of R, and I a non-zero ideal of R. If for any $x, y \in I$, there exists $n = n(x,y) \geq 1$ such that $(d([x, y]) - [x, y])^n = 0$, then R is commutative. It