Communications in Mathematical Research
30(4)(2014), 295–300

A Weak Convergence Theorem for A Finite Family of Asymptotically Nonexpansive Mappings

Kan Xu-zhou and Guo Wei-ping
(*School of Mathematics and Physics, Suzhou University of Science and Technology, Suzhou, Jiangsu, 215009*)

Communicated by Ji You-qing

Abstract: The purpose of this paper is to prove a new weak convergence theorem for a finite family of asymptotically nonexpansive mappings in uniformly convex Banach space.

Key words: asymptotically nonexpansive mapping, weak convergence, common fixed point, uniformly convex Banach space
2010 MR subject classification: 47H09, 47H10
Document code: A
Article ID: 1674-5647(2014)04-0295-06
DOI: 10.13447/j.1674-5647.2014.04.02

1 Introduction and Preliminaries

Throughout this paper, we assume that E is a real Banach space, E^* is the dual space of E and $J : E \to 2^{E^*}$ is the normalized duality mapping defined by

$$J(x) = \{f \in E^* : \langle x, f\rangle = \|x\|\|f\|,\ \|f\| = \|x\|\}, \qquad x \in E,$$

where $\langle\ \cdot\ ,\ \cdot\ \rangle$ denotes duality pairing between E and E^*. A single-valued normalized duality mapping is denoted by j.

Let K be a nonempty subset of E and $T : K \to K$ be a mapping. T is said to be asymptotically nonexpansive (see [1]) if there exists a sequence $\{h_n\} \subset [1, \infty)$ with $\lim\limits_{n\to\infty} h_n = 1$ such that

$$\|T^n x - T^n y\| \le h_n\|x - y\|, \qquad x, y \in K,\ n \ge 1.$$

Proposition 1.1[2] *Let K be a nonempty subset of E, and $\{T_i\}_{i=1}^N : K \to K$ be N asymptotically nonexpansive mappings. Then there exists a sequence $\{h_n\} \subset [1, \infty)$ with $h_n \to 1$*

Received date: March 31, 2012.
Foundation item: The NSF (11271282) of China.
E-mail address: kanxuzhou925@126.com (Kan X Z).

such that

$$\|T_i^n x - T_i^n y\| \le h_n \|x - y\|, \qquad n \ge 1,\ x, y \in K,\ i = 1, 2, \cdots, N. \tag{1.1}$$

Let K be a nonempty closed convex subset of E, $x_0 \in K$ be any given point and $\{T_i\}_{i=1}^N : K \to K$ be N asymptotically nonexpansive mappings. Let $\{h_n\}$ be the sequence defined by (1.1) and $L = \sup\limits_{n\ge1} h_n$. Then the sequence $\{x_n\} \subset K$ defined by

$$\begin{cases} x_1 = \alpha_1 x_0 + (1-\alpha_1) T_1 x_1, \\ x_2 = \alpha_2 x_1 + (1-\alpha_2) T_2 x_2, \\ \qquad\vdots \\ x_N = \alpha_N x_{N-1} + (1-\alpha_N) T_N x_N, \\ x_{N+1} = \alpha_{N+1} x_N + (1-\alpha_{N+1}) T_1^2 x_{N+1}, \\ \qquad\vdots \\ x_{2N} = \alpha_{2N} x_{2N-1} + (1-\alpha_{2N}) T_N^2 x_{2N}, \\ x_{2N+1} = \alpha_{2N+1} x_{2N} + (1-\alpha_{2N+1}) T_1^3 x_{2N+1}, \\ \qquad\vdots \end{cases} \tag{1.2}$$

is called the implicit iterative sequence for a finite family of asymptotically nonexpansive mappings $\{T_1, T_2, \cdots, T_N\}$, where $\tau \le \alpha_n \le 1 - \tau < \dfrac{1}{L}$ for all $n \ge 1$ and some $\tau > 0$.

Note that each $n \ge 1$ can be written as $n = (k-1)N + i$, where $i = i(n) \in \{1, 2, \cdots, N\}$, and $k = k(n) \ge 1$ is a positive integer with $k(n) \to \infty$ as $n \to \infty$. Hence we can write (1.2) in the following compact form:

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n) T_{i(n)}^{k(n)} x_n, \qquad n \ge 1. \tag{1.3}$$

In 2006, Chang *et al.*[2] studied the iteration process (1.3) and proved the following weak convergence theorem.

Theorem 1.1 *Let E be a real uniformly convex Banach space satisfying Opial condition, K be a nonempty closed convex subset of E, $\{T_1, T_2, \cdots, T_N\} : K \to K$ be N asymptotically nonexpansive mappings with $F = \bigcap\limits_{i=1}^N F(T_i) \ne \emptyset$, $\{\alpha_n\}$ be a sequence in $[0, 1]$ and $\{h_n\}$ be the sequence defined by (1.1) with $L = \sup\limits_{n\ge1} h_n \ge 1$ satisfying the following conditions:*

(i) $\sum\limits_{n=1}^{\infty} (h_n - 1) < \infty$;

(ii) *There exist constants $\tau_1, \tau_2 \in (0, 1)$ such that*

$$\tau_1 \le 1 - \alpha_n \le \tau_2, \qquad n \ge 1.$$

Then the implicit iterative sequence $\{x_n\}$ defined by (1.3) converges weakly to a common fixed point of $\{T_1, T_2, \cdots, T_N\}$ in K.

Only Theorem 1.1 has been obtained from the weak convergence problem for the sequence defined by (1.3). The purpose of this paper is to prove a new weak convergence theorem of the iteration scheme (1.3) for N asymptotically nonexpansive mappings in a uniformly convex Banach space.