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Existence Results for Periodic Solutions of Nonautonomous Second-order Differential Systems with (q, p)-Laplacian*

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Abstract: In this paper, we consider the existence for periodic solutions of nonautonomous second-order differential systems with (q, p)-Laplacian by using the least action principle and the minimax methods.

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1 Introduction and Main Results

Consider the second-order system

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}(|\dot{u}_1(t)|^{q-2}\dot{u}_1(t)) = \nabla_{u_1} F(t, u_1(t), u_2(t)), \\ \dfrac{\mathrm{d}}{\mathrm{d}t}(|\dot{u}_2(t)|^{p-2}\dot{u}_2(t)) = \nabla_{u_2} F(t, u_1(t), u_2(t)), \\ u_1(0) = u_1(T), \qquad \dot{u}_1(0) = \dot{u}_1(T), \\ u_2(0) = u_1(T), \qquad \dot{u}_2(0) = \dot{u}_2(T), \end{cases} \tag{1.1}$$

where $T > 0$, $1 < q, p < \infty$, and $F : [0, T] \times \mathbf{R}^N \times \mathbf{R}^N \to \mathbf{R}^1$ satisfies the following assumptions:

(A) F is measurable in t for each $(x_1, x_2) \in \mathbf{R}^N \times \mathbf{R}^N$, continuously differentiable in (x_1, x_2) for $t \in [0, T]$ a.e. and there exist $a_1, a_2 \in C(\mathbf{R}^+, \mathbf{R}^+)$ and $b \in L^1(0, T; \mathbf{R}^+)$ such that

$$|F(t, x_1, x_2)| + |\nabla_{x_1} F(t, x_1, x_2)| + |\nabla_{x_2} F(t, x_1, x_2)| \leq (a_1(|x_1|) + a_2(|x_2|))b(t)$$

for all $(x_1, x_2) \in \mathbf{R}^N \times \mathbf{R}^N$ and $t \in [0, T]$ a.e.

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Denote by $\langle\,\cdot\,,\,\cdot\,\rangle$ and $|\cdot|$ the inner product and the norm of $\mathbf{R}^n$, respectively. The corresponding functional $\varphi : W \to \mathbf{R}$ given by

$$\varphi(u_1, u_2) = \frac{1}{q}\int_0^T |\dot{u}_1|^q \mathrm{d}t + \frac{1}{p}\int_0^T |\dot{u}_2|^p \mathrm{d}t + \int_0^T F(t, u_1(t), u_2(t))\mathrm{d}t$$

is continuously differentiable on W and

$$\begin{aligned}\langle\varphi'(u_1, u_2),\ (v_1, v_2)\rangle = &\int_0^T [(|\dot{u}_1(t)|^{q-2}\dot{u}_1(t),\ \dot{v}_1) + (\nabla_{x_1} F(t, u_1, u_2),\ v_1)]\mathrm{d}t\\ &+ \int_0^T [(|\dot{u}_2(t)|^{p-2}\dot{u}_2(t),\ \dot{v}_2) + (\nabla_{x_2} F(t, u_1, u_2),\ v_2)]\mathrm{d}t\end{aligned} \tag{1.2}$$

for all $(u_1, u_2), (v_1, v_2) \in W$, where

$$W = W_T^{1,q} \times W_T^{1,p}$$

is a reflexive Banach space with the norm

$$\|(u_1, u_2)\|_W = \|u_1\|_{W_T^{1,q}} + \|u_2\|_{W_T^{1,p}}.$$

Moreover, the solutions of the problem (1.1) correspond to the critical points of φ (see [1–2]).

For each $u \in W_T^{1,p}$, it can be written as

$$u(t) = \bar{u} + \tilde{u}(t),$$

where

$$\bar{u} = \frac{1}{T}\int_0^T u(t)\mathrm{d}t,$$

$$\int_0^T \tilde{u}(t)\mathrm{d}t = 0.$$

Then we have Sobolev's ineuqality (see [3]):

$$\|\tilde{u}\|_\infty \le C_1\|\dot{u}\|_q, \quad \|\tilde{v}\|_\infty \le C_1\|\dot{v}\|_p, \qquad u \in W_T^{1,q},\ v \in W_T^{1,p},$$

and Wirtinger's inequality (see [3]):

$$\|\tilde{u}\|_q \le C_2\|\dot{u}\|_q, \quad \|\tilde{v}\|_p \le C_2\|\dot{v}\|_p, \qquad u \in W_T^{1,q}, v \in W_T^{1,p}$$

for some positive constants C_1 and C_2, where

$$\|u\|_p = \Big(\int_0^T |u(t)|^p \mathrm{d}t\Big)^{\frac{1}{p}},$$

$$\|u\|_\infty = \max_{t\in[0,T]} |u(t)|.$$

The existence of periodic solutions of second-order systems has been extensively studied and a lot of important existence results have been obtained, for example, see [4–6] and the references therein. There are also some papers (see [7–9]) on the periodic solutions of second-order systems with a p-Laplacian, in which a lot of results on Hamiltonian systems are generalized.

Paşca and Tang[1] proved the existence results for the problem (1.1). In this paper we continue to consider the problem (1.1) with some new solvability conditions by using the least action principle and saddle point theorem.

We state the main results of this paper:

Theorem 1.1 *Let q' and p' be positive constants such that*

$$\frac{1}{q} + \frac{1}{p'} = 1, \qquad \frac{1}{q} + \frac{1}{q'} = 1.$$