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Linearly McCoy Rings and Their Generalizations*

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Abstract: A ring R is called linearly McCoy if whenever linear polynomials $f(x)$, $g(x) \in R[x]\backslash\{0\}$ satisfy $f(x)g(x) = 0$, then there exist nonzero elements r, $s \in R$ such that $f(x)r = sg(x) = 0$. For a ring endomorphism α, we introduced the notion of α-skew linearly McCoy rings by considering the polynomials in the skew polynomial ring $R[x;\ \alpha]$ in place of the ring $R[x]$. A number of properties of this generalization are established and extension properties of α-skew linearly McCoy rings are given.
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1 Introduction

Throughout the paper, R denotes an associative ring with identity 1. The notation α denotes an endomorphism of a given ring, and $\alpha(1)$ need not be equal to 1 in this paper.

McCoy[1] proved in 1942 that if two polynomials annihilate each other over a commutative ring, then each polynomial has a nonzero annihilator in the base ring. Rege and Chhawchharia[2] and Nielsen[3] introduced the notion of a McCoy ring, independently. Recall that a ring R is called right McCoy if the equation

$$f(x)g(x) = 0$$

with nonzero $f(x)$, $g(x) \in R[x]$, implies that there exists a nonzero $r \in R$ such that

$$f(x)r = 0.$$

Left McCoy rings are defined similarly. A ring R is said to be McCoy if it is both right and left McCoy. The concept of right (left) linearly McCoy rings were introduced by Camillo and Nielsen[4] in 2008. A linearly McCoy ring is both left and right linearly McCoy. It was

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proved in [4] that semi-commutative rings (i.e., $ab = 0$ implies $aRb = 0$ for $a, b \in R$) are linearly McCoy, but the converse is not true.

For a ring R with a ring endomorphism $\alpha : R \to R$, a skew polynomial ring $R[x;\ \alpha]$ of R is the ring obtained by giving the polynomial ring over R with the new multiplication

$$xr = \alpha(r)x, \qquad \forall r \in R.$$

Due to Hong *et al.*[5], the Armendariz property of a ring was extended to skew polynomial rings but with skewed scalar multiplication: For an endomorphism α of a ring R, R is called α-skew Armendariz if

$$\Big(\sum_{i=0}^{m} a_i x^i\Big)\Big(\sum_{j=0}^{n} b_j x^j\Big) = 0 \qquad \text{in } R[x;\ \alpha]$$

implies

$$a_i \alpha^i(b_j) = 0 \qquad \text{for all } 0 \le i \le m \text{ and } 0 \le j \le n$$

(R is called Armendariz if $\alpha = I_R$). And Lei[6] extended McCoy rings to α-skew McCoy rings.

Motivated by the above, we introduced the notion of an α-skew linearly McCoy ring with the endomorphism α, as both a generalization of α-skew McCoy rings and an extension of linearly McCoy rings. The properties of this class of rings are investigated.

2 Linearly McCoy Rings

Our focus in this section is to discuss the basic properties of linearly McCoy rings and observe the connections to other related rings; extension properties of the rings are investigated.

Lemma 2.1 *A ring R is right* (*resp., left*) *linearly McCoy if and only if the ring*

$$V(R) = \left\{ \begin{pmatrix} a_1 & u_1 & 0 & 0 & 0 & 0 \\ 0 & a_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & a_3 & u_2 & 0 & 0 \\ 0 & 0 & 0 & a_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a_2 & u_3 \\ 0 & 0 & 0 & 0 & 0 & a_3 \end{pmatrix} : a_1, a_2, a_3, u_1, u_2, u_3 \in R \right\}$$

is right (*resp., left*) *linearly McCoy.*

Proof. It suffices to prove the case when R is right linearly McCoy.

For any A, $B \in V(R)$, write

$$A = (a_1,\ a_2,\ a_3,\ u_1,\ u_2,\ u_3), \qquad B = (b_1,\ b_2,\ b_3,\ v_1,\ v_2,\ v_3).$$

The addition and multiplication in $V(R)$ are defined as follows:

$$A + B = (a_1 + b_1,\ a_2 + b_2,\ a_3 + b_3,\ u_1 + v_1,\ u_2 + v_2,\ u_3 + v_3),$$

$$AB = (a_1b_1,\ a_2b_2,\ a_3b_3,\ a_1v_1 + u_1b_2,\ a_3v_2 + u_2b_1,\ a_2v_3 + u_3b_3).$$

"$\Rightarrow$". Let

$$F(x) = A_0 + A_1x,\ G(x) = B_0 + B_1x \in V(R)[x] \setminus \{0\}$$