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Convergence Properties of Generalized Fourier Series on a Parallel Hexagon Domain*

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Abstract: A new Rogosinski-type kernel function is constructed using kernel function of partial sums $S_n(f;t)$ of generalized Fourier series on a parallel hexagon domain Ω associating with three-direction partition. We prove that an operator $W_n(f;t)$ with the new kernel function converges uniformly to any continuous function $f(t) \in C_*(\Omega)$ (the space of all continuous functions with period Ω) on Ω. Moreover, the convergence order of the operator is presented for the smooth approached function.

Key words: three-direction coordinate, kernel function, generalized Fourier series, uniform convergence

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1 Introduction

It is well known that Fourier methods play a key role in numerical analysis and its applications. There has been a lot of work relating to Fourier series (see [1]). As we know, the original results are first studied in the univariate case and then generalized into multivariate cases in a high dimension by techniques of tensor product. Strictly, the tensor product approach is still staying in the one dimensional level via decreasing dimension. The approach is only suitable for rectangular domains in 2-dimension case. Therefore, researchers are now paying more and more attention to the studies on how to generalize Fourier methods into high dimension, beyond box domains. In 2003, Sun[2] proposed Fourier methods on parallel hexagon domain associating with three-direction partition. It has been pointed that the most concepts and results of Fourier methods on tensor-product case can be moved on non tensor-product case. Reference [3] presented Cubature formula and interpolation on parallel

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hexagon domain, in which Lebesgue constant of the interpolation polynomial is shown to be $O[(\log n)^2]$. Reference [2] proved that the second order partial sum of Fourier series of $f(t) \in C(\Omega)$ (the space of all continuous functions $f(t)$ on Ω) converges to $f(t)$. But, theorem 3.5 of [2] shows that the first partial sum of Fourier series of $f(t) \in C(\Omega)$ can't converge uniformly to $f(t)$. In view of this, a new Rogosinski-type kernel function is constructed using the kernel function of partial sums $S_n(f;t)$ of generalized Fourier series in [2]. The operator $W_n(f;t)$ associating with the new kernel function converges uniformly on Ω.

In $\mathbf{R}^2$, take $e_1 = (1,0)$, $e_2 = \Big(-\frac{1}{2}, \frac{\sqrt{3}}{2}\Big)$, and $e_3 = \Big(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\Big)$. Define the normal vectors n_1, n_2, n_3 of e_1, e_2, e_3, respectively:

$$n_1 = e_2 - \frac{(e_2,e_1)}{(e_1,e_1)}e_1, \qquad n_2 = e_3 - \frac{(e_3,e_2)}{(e_2,e_2)}e_2, \qquad n_3 = e_1 - \frac{(e_1,e_3)}{(e_3,e_3)}e_1.$$

And then, a new three-direction coordinate system $\widetilde{\mathbf{R}}^2 = O(n_1,n_2,n_3)$ is set up as follows: an one-to-one correspondence is established between the point $t = (t_1,t_2,t_3)$ under a three-direction coordinate system and the point $x = (x_1,x_2) \in \mathbf{R}^2$ under an original Cartesian coordinate system (see Fig. 1.1), where

$$t_1 = \frac{(x,n_1)}{(n_1,n_1)}, \qquad t_2 = \frac{(x,n_2)}{(n_2,n_2)}, \qquad t_3 = \frac{(x,n_3)}{(n_3,n_3)}.$$

Namely,

$$t_1 = \frac{2\sqrt{3}}{3}x_2, \qquad t_2 = -x_1 - \frac{\sqrt{3}}{3}x_2, \qquad t_3 = x_1 - \frac{\sqrt{3}}{3}x_2.$$

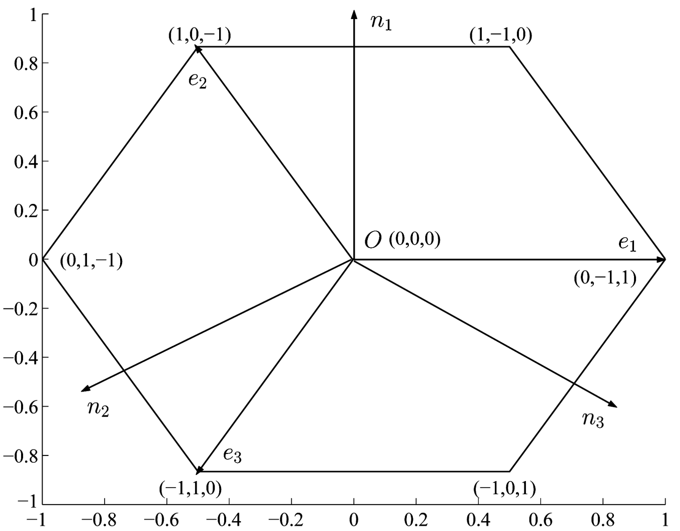

Fig. 1.1 3-direction coordinates

It is easy to verify that the three-direction coordinate satisfies the following identity

$$t_1 + t_2 + t_3 = 0.$$

We take the following parallel hexagon, drawn in Fig. 1.1, as our basic domain

$$\Omega = \{t = (t_1,t_2,t_3) \in \widetilde{\mathbf{R}}^2 | -1 \le t_1,t_2,t_3 \le 1,\ t_1+t_2+t_3 = 0,\ t_1,t_2,t_3 \in \mathbf{R}\}.$$