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Blow-up vs. Global Finiteness for an Evolution p-Laplace System with Nonlinear Boundary Conditions*

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Abstract: In this paper, the authors consider the positive solutions of the system of the evolution p-Laplacian equations

$$\begin{cases} u_t = \mathrm{div}(| \nabla u |^{p-2} \nabla u) + f(u, v), & (x, t) \in \Omega \times (0, T), \\ v_t = \mathrm{div}(| \nabla v |^{p-2} \nabla v) + g(u, v), & (x, t) \in \Omega \times (0, T) \end{cases}$$

with nonlinear boundary conditions

$$\frac{\partial u}{\partial \eta} = h(u, v), \qquad \frac{\partial v}{\partial \eta} = s(u, v),$$

and the initial data $(u_0,\ v_0)$, where Ω is a bounded domain in $\mathbf{R}^n$ with smooth boundary $\partial\Omega$, $p > 2$, $h(\,\cdot\,,\,\cdot\,)$ and $s(\,\cdot\,,\,\cdot\,)$ are positive C^1 functions, nondecreasing in each variable. The authors find conditions on the functions f, g, h, s that prove the global existence or finite time blow-up of positive solutions for every (u_0, v_0).

Key words: nonlinear boundary value problem, evolution p-Laplace system, blow-up

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1 Introduction

In this paper, we consider the system

$$\begin{cases} u_t = \mathrm{div}(|\nabla u|^{p-2}\nabla u) + f(u, v), & (x, t) \in \Omega \times (0, T), \\ v_t = \mathrm{div}(|\nabla v|^{p-2}\nabla v) + g(u, v), & (x, t) \in \Omega \times (0, T) \end{cases} \tag{1.1}$$

with nonlinear boundary conditions

$$\begin{cases} \dfrac{\partial u}{\partial \eta} = h(u, v), & (x, t) \in \partial\Omega \times (0, T), \\ \dfrac{\partial v}{\partial \eta} = s(u, v), & (x, t) \in \partial\Omega \times (0, T), \end{cases} \tag{1.2}$$

where $p > 2$, Ω is a bounded domain in $\mathbf{R}^n$ with smooth boundary $\partial\Omega$, $f(\,\cdot\,,\,\cdot\,)$ and $g(\,\cdot\,,\,\cdot\,)$ are both nonnegative continuous functions and nondecreasing in each variable, and $h(\,\cdot\,,\,\cdot\,)$

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and $s(\cdot\,,\cdot)$ are both positive C^1 functions and nondecreasing in each variable. The initial data are

$$\begin{cases} u(x,0)=u_0(x), & x\in\Omega,\\ v(x,0)=v_0(x), & x\in\Omega, \end{cases} \tag{1.3}$$

where u_0, v_0 are positive continuous functions on $\overline{\Omega}$.

When $p=2$, the problem

$$\begin{cases} u_t=\triangle u, & (x,t)\in\Omega\times(0,T),\\ \dfrac{\partial u}{\partial\eta}=f(u), & (x,t)\in\partial\Omega\times(0,T),\\ u(x,0)=u_0(x), & x\in\Omega \end{cases}$$

has been studied by many authors (see [1]–[5]). In [1], Rial and Rossi proved a blow-up result under the condition

$$\int^{+\infty}\frac{1}{f}<+\infty.$$

In [2], Walter proved a global existence result. In [3]–[5], the authors obtained some results on local existence of classical or weak solutions.

With the use of supersolution-subsolution method, we relate (1.1)–(1.3) to the corresponding system of nonlinear differential equations

$$\begin{cases} \varphi'(\sigma)=h(\varphi(\sigma),\psi(\sigma)), & \sigma\in R,\\ \psi'(\sigma)=s(\varphi(\sigma),\psi(\sigma)), & \sigma\in R,\\ \varphi(0)=\varphi_0,\\ \psi(0)=\psi_0, \end{cases} \tag{1.4}$$

where φ_0, ψ_0 are suitable nonegative constants. By constructing a subsolution or a supersolution, we can obtain the global finiteness or blow-up properties to the positive solutions of the system respectively. The similar ideas can be found in [6] and [7]. We obtain the main results as follows.

Theorem 1.1 *If the positive solution of* (1.4) *blows up, then the positive solution of* (1.1)–(1.3) *blows up.*

Suppose that (1.4) has a global positive solution $(\varphi(\sigma),\psi(\sigma))$. Set

$$\begin{cases} F(\sigma)=(\varphi'(\sigma))^{p-2}\varphi''(\sigma)+(\varphi'(\sigma))^{p-1}+f(\varphi(\sigma),\psi(\sigma)), & \sigma\in R,\\ G(\sigma)=(\psi'(\sigma))^{p-2}\psi''(\sigma)+(\psi'(\sigma))^{p-1}+g(\varphi(\sigma),\psi(\sigma)), & \sigma\in R. \end{cases}$$

And suppose that $\dfrac{F(\sigma)}{\varphi'(\sigma)}$, $\dfrac{G(\sigma)}{\psi'(\sigma)}$ are monotonically increasing or decreasing simultaneously. We get the following theorems.

Theorem 1.2 *If*

$$\int^{\infty}\frac{1}{\min\left\{\dfrac{F(\sigma)}{\varphi'(\sigma)},\dfrac{G(\sigma)}{\psi'(\sigma)}\right\}}\mathrm{d}\sigma<+\infty,$$

then the positive solution of (1.1)–(1.3) *blows up.*