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Ricci Curvature of Certain Submanifolds in Kenmotsu Space Forms*

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Abstract: In this paper, we obtain some sharp inequalities between the Ricci curvature and the squared mean curvature for bi-slant and semi-slant submanifolds in Kenmotsu space forms. Estimates of the scalar curvature and the k-Ricci curvature, in terms of the squared mean curvature, are also proved respectively.

Key words: Kenmotsu space form, Ricci curvature, k-Ricci curvature, bi-slant submanifold, semi-slant submanifold

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1 Introduction

One of the basic problems in submanifold theory is to find some relationships between the main extrinsic invariants and the main intrinsic invariants of a submanifold. Scalar curvature and Ricci curvature are the main intrinsic invariants, while the squared mean curvature is the main extrinsic invariant. In [1], Chen established a relationship between sectional curvature and the shape operator for submanifolds in real space forms. In [2], he also got a relationship between Ricci curvature and squared mean curvature.

A contact version of Chen's inequality and its applications to slant immersions in a Sasakian space form $\widetilde{M}(c)$ were given in [3]. There is another interesting class of almost contact metric manifolds, namely Kenmotsu manifolds (see [4]). In the present paper, we study the Ricci curvature of certain submanifolds in a Kenmotsu space form, and get some very interesting results. The rest of this paper is organized as follows. Necessary details about Kenmotsu manifolds and the submanifolds of Kenmotsu manifolds are reviewed in Section 2. In Section 3, we prove some inequalities between Ricci curvature and squared mean curvature function for bi-slant and semi-slant submanifolds in Kenmotsu space forms.

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In the last section, we establish some relationship between the k-Ricci curvature and the squared mean curvature for bi-slant and semi-slant submanifolds in Kenmotsu space forms.

2 Preliminaries

Let $\widetilde{M}$ be an almost contact metric manifold with an almost contact metric structure (ϕ, ξ, η, g) (see [5]). That is, there exist a $(1,1)$ tensor field ϕ, a vector field ξ, a 1-form η and a Riemannian metric g on $\widetilde{M}$, such that for all $X, Y \in T\widetilde{M}$ one has

$$\phi^2 = -I + \eta \otimes \xi, \qquad \eta(\xi) = 1, \qquad \phi(\xi) = 0, \qquad \eta \circ \phi = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.2}$$

$$g(X, \phi Y) = -g(\phi X, Y), \qquad g(X, \xi) = \eta(X). \tag{2.3}$$

An almost contact metric manifold is a Kenmotsu manifold (see [4]) if

$$(\widetilde{\nabla}_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{2.4}$$

$$\widetilde{\nabla}_X \xi = -\phi^2 X = X - \eta(X)\xi, \qquad X \in T\widetilde{M} \tag{2.5}$$

for any vector fields X, Y on $\widetilde{M}$, where $\widetilde{\nabla}$ denotes the Riemannian connection on $\widetilde{M}$.

We denote by Φ the fundamental 2-form, that is,

$$\Phi(X, Y) = g(\varphi X, Y)$$

for any vector fields X, Y on $\widetilde{M}$. It was known that the pairing (Φ, η) defines a locally conformal cosymplectic structure, that is,

$$d\Phi = 2\Phi \wedge \eta, \qquad d\eta = 0. \tag{2.6}$$

A plane section σ in $T_p\widetilde{M}$ of an almost contact metric manifold $\widetilde{M}$ is called a φ-section if $\sigma \perp \xi$ and $\varphi(\sigma) = \sigma$. $\widetilde{M}$ is of constant φ-sectional curvature if at each point $p \in \widetilde{M}$, the sectional curvature $\widetilde{K}(\sigma)$ does not depend on the choice of the φ-section σ of $T_p\widetilde{M}$, and in this case for $p \in \widetilde{M}$ and for any φ-section σ of $T_p\widetilde{M}$, the function c defined by $c(p) = \widetilde{K}(\sigma)$ is called the φ-sectional curvature of $\widetilde{M}$. A Kenmotsu manifold $\widetilde{M}$ with constant φ-sectional curvature c is said to be a Kenmotsu space form and is denoted by $\widetilde{M}(c)$.

The curvature tensor $\widetilde{R}$ of a Kenmotsu space form $\widetilde{M}(c)$ is given by (see [4])

$$\begin{aligned}
&4\widetilde{R}(X, Y, Z, W) \\
&= (c-3)[g(X, W)g(Y, Z) - g(X, Z)g(Y, W)] + (c+1)[g(\varphi X, W)g(\varphi Y, Z) \\
&\quad - g(\varphi X, Z)g(\varphi Y, W) - 2g(\varphi X, Y)g(\varphi Z, W) + g(X, Z)\eta(Y)\eta(W) \\
&\quad - g(Y, Z)\eta(X)\eta(W) + g(Y, W)\eta(X)\eta(Z) - g(X, W)\eta(Y)\eta(Z)]
\end{aligned} \tag{2.7}$$

for all X, Y, Z, $W \in T\widetilde{M}$.

Let M be an n-dimensional Riemannian manifold. The scalar curvature τ at p is given by

$$\tau = \sum_{i<j} K_{ij},$$

where K_{ij} is the sectional curvature of M associated with a plane section spanned by e_i and e_j at $p \in M$ for any orthonormal basis $\{e_1, \cdots, e_n\}$ for T_pM. Now let M be a submanifold