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A General Non-Lipschitz Joint Regularized Model for Multi-Channel/Modality Image Reconstruction

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Abstract. Multi-channel/modality image joint reconstruction has gained much research interest in recent years. In this paper, we propose to use a nonconvex and non-Lipschitz joint regularizer in a general variational model for joint reconstruction under additive measurement noise. This framework has good ability in edge-preserving by sharing common edge features of individual images. We study the lower bound theory for the non-Lipschitz joint reconstruction model in two important cases with Gaussian and impulsive measurement noise, respectively. In addition, we extend previous works to propose an inexact iterative support shrinking algorithm with proximal linearization for multi-channel image reconstruction (InISSAPL-MC) and prove that the iterative sequence converges globally to a critical point of the original objective function. In a special case of single channel image restoration, the convergence result improves those in the literature. For numerical implementation, we adopt primal dual method to the inner subproblem. Numerical experiments in color image restoration and two-modality undersampled magnetic resonance imaging (MRI) reconstruction show that the proposed non-Lipschitz joint reconstruction method achieves considerable improvements in terms of edge preservation for piecewise constant images compared to existing methods.

AMS subject classifications: 68U10, 65K10, 94A08, 90C26

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1 Introduction

In recent years, multi-channel/modality image reconstruction has attracted a lot of attention in inverse problems, such as color image restoration [9,27,52,54] and joint reconstruction in medical imaging [20,21,44,59]. Most of them aim to exploit similar structures from multiple channels to improve the reconstruction quality. Let $u=(u^1,\cdots,u^M)$ be the

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M-channel/modality images. A general variational framework for joint image reconstruction is given by

$$\min_u R(u) + \sum_{m=1}^{M} F_m(u^m), \tag{1.1}$$

where $F_m(u^m)$ refers to the fidelity term associated with the data observation of each channel/modality, and $R(u)$ is called regularization term which corresponds to a priori knowledge combination of images. A good regularization should find useful complementary features across different channels or modalities while preserving the unique features within each channel/modality and avoiding cross-interaction artifacts. Therefore, the choice of $R(u)$ is crucial in joint reconstruction problems. In general, the main structures shared by different channels are the common edges featured by gradient information, such as the nonconvex parallel level sets functional [21], the nonconvex edge weighted total variation (TV) [59] and the convex joint TV [29], as well as the joint sparsity [16,48].

In this paper, we focus on the variational setting for the joint reconstruction problem. Regarding the edge-preserving property, a class of general and classical multi-channel/modality regularizers are extended from TV [46], which has been successfully applied in [4,5,8,33,50,61]. The first known extension to color images was proposed by Blomgren and Chan [6]. They applied the usual ℓ_2 norm to the vector of TV seminorms of all channels, that is, $\mathrm{CTV}(u) = \sqrt{\sum_{m=1}^{M} \mathrm{TV}(u^m)}$. This regularizer computes the TV of each channel and then combines them together. Another general and widely used vectorial TV is the joint TV (JTV) taking the form of

$$\mathrm{JTV}(u) = \int_\Omega \sqrt{\sum_{m=1}^{M} |\nabla u^m|^2 \mathrm{d}x}, \tag{1.2}$$

which locally combines the gradients of all channels at each pixel with an ℓ_2 norm coupling. The JTV favors sparsity in the magnitude of the 'joint gradient', i.e., $\nabla u = (\nabla u^1, \cdots, \nabla u^M)$. It has been successfully used in color image restoration [9,52,54], multi-modality MRI reconstruction [15, 32] and PET-MRI joint reconstruction [21, 39]. Also, there are some other vectorial TVs defined with respect to geometric structure; see, for instance, [27,36]. We refer the reader to [19] for detailed insights of the various vectorial TVs mentioned above.

Recently, nonconvex and nonsmooth composite minimization models have been shown to possess good ability in recovering neat edges [14,30,41,42]. These findings and observations are supported by the lower bound theory of nonconvex models; see, for instance, [14, 40, 56]. In these models, the regularizers are usually constructed by some concave potential functions composed of the gradient operator. However, so far, such nonconvex regularizations have been only applied in single-channel problems. Considering that the crucial point in joint reconstruction is to recover their common edges of multi-channel/modality images, we are therefore motivated to compose a nonconvex